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value of x, we get a single definite value of y. Hence $y = 3x^2+7x+2$ represents a function of x.

LIMIT OF A FUNCTION

A function $f(x)$ is said to have a limit l as $x \to a$ (read as x approaches a) if the difference $|f(x)-l|$ is negligibly small but not equals to zero. It is represented by

$$\lim_{x \to a} f(x) = l$$

e.g let us consider a function,

$$y = f(x) = \frac{x^3-8}{x-2}$$

This function is defined at all points except at $x=2$, because at $x=2$,

$$y = f(x) = \frac{2^3-8}{2-2} = \frac{0}{0} = \infty$$

But for all the values of x very close to 2 (i.e more or less than 2) the function tends to 12

i.e when $x = 1.98$, $f(x) = \frac{(1.98)^3-8}{1.98-2} = 11.88$

when $x = 1.99$, $f(x) = \frac{(1.99)^3-8}{1.99-2} = 11.94$

when $x = 2.01$, $f(x) = \frac{(2.01)^3-8}{2.01-2} = 12.06$

when $x = 2.02$, $f(x) = \frac{(2.02)^3-8}{2.02-2} = 12.12$

Thus for all values of x very close to 2, but not equal to 2, $\frac{x^3-8}{x-2} \approx 12$.

Examples:- Evaluate (i) $\lim_{x\to 2} (6x^2+4x)$

Sol:- $\lim_{x\to 2} (6x^2+4x) = \lim_{x\to 2}(6x^2) + \lim_{x\to 2}(4x)$

$= 6(2)^2+4(2) = 24+8 = 32$.

(ii) $\lim_{x\to 2} \frac{x^2-4}{x-2}$

Solution $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x)^2-(2)^2}{(x-2)} = \lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

$= \lim_{x\to 2} (x+2) = (2+2) = 4$.

(3) $\lim_{x\to \infty} \frac{3x^2+4x+5}{2x^2+6x+7}$

$= \lim_{x\to \infty} \frac{3x^2+4x+5}{2x^2+6x+7}$ Dividing by x^2

$= \lim_{x\to\infty} \frac{3+\frac{4}{x}+5/x^2}{2+\frac{6}{x}+7/x^2} = \frac{3+\frac{4}{\infty}+\frac{5}{\infty^2}}{2+6/\infty+7/\infty^2} = \frac{3+0+0}{2+0+0}$

$= 3/2$.

Note:- $\lim_{\theta\to 0} \sin\theta = 0$ (2) $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

(3) $\lim_{\theta\to 0} \cos\theta = 1$ (4) $\lim_{\theta\to 0} \tan\theta = 0$ (5) $\lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1$

(7) $\lim_{h\to 0} \frac{a^h-1}{h} = \log a$

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Q Evaluate $\lim_{x\to 0} \frac{\sin 9x}{\tan 6x}$

$$= \frac{\lim_{x\to 0} \frac{\sin 9x}{9x} \cdot 9x}{\lim_{x\to 0} \frac{\tan 6x}{6x} \cdot 6x} \quad \left[\because \lim_{x\to 0} \frac{\sin\theta}{\theta} = 1 \text{ \& } \lim_{x\to 0} \frac{\tan\theta}{\theta} = 1\right]$$

$$= \frac{1\times 9}{1\times 6} = \frac{\cancel{9}\,3}{\cancel{6}\,2} = 3/2$$

Differentiation

The process of finding differential coefficient or derivative of a function with respect to (w.r.t) a variable is called differentiation.

Derivative OR Differential coefficient:-> The derivative or differential coefficient of a function is defined as the instantaneous rate of change of the function.

In order to understand the concept of derivative, let us consider that y is a function of x. i.e

$$y = f(x) \quad \text{--- (1)}$$

Let Δx = small increase in the value of x

and Δy = small increase in the value of y, Then

$\frac{\Delta y}{\Delta x}$ = Quotient of increments or differances.

The limit of $\frac{\Delta y}{\Delta x}$ as $\Delta x \to 0$ is called the derivative or differential coefficient of y w.r.t x. It is denoted by $\frac{dy}{dx}$ i.e

$$\lim_{\Delta x\to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} \quad \text{--- (2)}$$