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or $\Delta y = [(ax+b)+a\Delta x]^n - (ax+b)^n$

or $\Delta y = (ax+b)^n\left[1+\frac{a\Delta x}{(ax+b)}\right]^n - (ax+b)^n$

or $\Delta y = (ax+b)^n\left[\left\{1+\frac{a\Delta x}{(ax+b)}\right\}^n - 1\right]$

using Bionomial theorem, we get

$$\Delta y = (ax+b)^n\left[\left\{1+\frac{na\Delta x}{(ax+b)}+\frac{n(n-1)}{2!}\left(\frac{a\Delta x}{(ax+b)}\right)^2 + \cdots\right\} - 1\right]$$

or $\Delta y = (ax+b)^n\left[\frac{na\Delta x}{(ax+b)}+\frac{n(n-1)}{2!}\left(\frac{a\Delta x}{ax+b}\right)^2+\cdots\right]$

Dividing b/s by Δx, we get

$$\frac{\Delta y}{\Delta x} = \frac{(ax+b)^n}{\Delta x}\left[\frac{na\Delta x}{(ax+b)}+\frac{n(n-1)}{2!}\left(\frac{a\Delta x}{ax+b}\right)^2+\cdots\right]$$

$$\frac{\Delta y}{\Delta x} = (ax+b)^n\left[\frac{na}{(ax+b)}+\frac{n(n-1)}{2!}\frac{a^2\Delta x}{(ax+b)^2}+\cdots\right]$$

taking $\lim_{\Delta x\to 0}$ to b/s we get

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}(ax+b)^n\left[\frac{na}{(ax+b)}+\frac{n(n-1)}{2!}\frac{a^2\Delta x}{(ax+b)}+\cdots\right]$$

or $\frac{dy}{dx} = (ax+b)^n\cdot\frac{na}{ax+b}+0$

or $\frac{dy}{dx} = (ax+b)^n\cdot\frac{na}{(ax+b)}+0$

or $\frac{dy}{dx} = na(ax+b)^{n-1}$ — (3)

using eq (1) in eq (3), we get

∴ Derivative or differential coefficient of $(ax+b)^n$ is $\frac{d}{dx}(ax+b)^n = na(ax+b)^{n-1}$

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(3) Prove That $\frac{d}{dx}(e^{ax}) = ae^{ax}$

Let $y = e^{ax}$ —(1)

Let Δx + Δy be the small increments in x and y respectively, then

$y + \Delta y = e^{a(x+\Delta x)}$ —(2)

Subtracting eqn (1) from eqn (2)

$y + \Delta y - y = e^{a(x+\Delta x)} - e^{ax}$

$\Delta y = e^{ax + a\Delta x} - e^{ax}$

or $\Delta y = e^{ax} \cdot e^{a\Delta x} - e^{ax}$

or $\Delta y = e^{ax}[e^{a\Delta x} - 1]$

Dividing b/s by Δx, we get

$$\frac{\Delta y}{\Delta x} = \frac{e^{ax}}{\Delta x}[e^{a\Delta x} - 1]$$

Taking limits $\Delta x \to 0$ to b/s we get

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{e^{ax}}{\Delta x}[e^{a\Delta x} - 1]$$

[Multiplying + dividing by a]

$$\text{or } \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{ae^{ax}[e^{a\Delta x} - 1]}{a\Delta x}$$

$$\text{or } \frac{dy}{dx} = ae^{ax} \lim_{\Delta x \to 0} \frac{[e^{a\Delta x} - 1]}{a\Delta x} \quad \text{— (3)}$$

As we know that

$$\lim_{\Delta x \to 0} \frac{e^{a\Delta x} - 1}{a\Delta x} = 1$$

$\therefore$ Eqn (3) becomes, $\frac{dy}{dx} = ae^{ax} \cdot 1$

Hence differentiation of e^{ax} is

$$\boxed{\frac{dy}{dx} = \frac{d}{dx} e^{ax} = ae^{ax}}$$

Hence Proved.

(4) Differentiation of sinx

Let $y = \sin x$ —(1)

Giving small increments Δx + Δy to x + y resp., we get

$y + \Delta y = \sin(x+\Delta x)$ —(2)

subtracting eqn (1) from (2)

$\therefore y + \Delta y - y = \sin(x+\Delta x) - \sin x$

$\Delta y = \sin(x+\Delta x) - \sin x$ —(3)

using the formula

i.e $\sin A - \sin B =$

$= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\therefore \Delta y = 2\cos\left(\frac{x+\Delta x+x}{2}\right) \times \sin\left(\frac{\cancel{x}+\Delta x-\cancel{x}}{2}\right)$

$= 2\cos\left(\frac{2x+\Delta x}{2}\right)\sin\frac{\Delta x}{2}$

or $\Delta y = 2\cos\left(x+\frac{\Delta x}{2}\right)\sin\frac{\Delta x}{2}$

Dividing b/s by Δx, we get

$\frac{\Delta y}{\Delta x} = \frac{2\cos\left(x+\frac{\Delta x}{2}\right)\sin\frac{\Delta x}{2}}{\Delta x}$

$= \frac{\cancel{2}\cos\left(x+\frac{\Delta x}{2}\right)\cdot\sin\frac{\Delta x}{2}}{\cancel{2}\,\frac{\Delta x}{2}}$

or $\frac{\Delta y}{\Delta x} = \cos\left(x+\frac{\Delta x}{2}\right)\cdot\frac{\sin\frac{\Delta x}{2}}{\Delta x/2}$

taking limits as $\Delta x \to 0$

$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0}\cos\left(x+\frac{\Delta x}{2}\right)\times\lim_{\Delta x\to 0}\frac{\sin\frac{\Delta x}{2}}{\Delta x/2}$

$\frac{dy}{dx} = \cos(x+0)\times 1$ $\left(\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right)$

$\therefore \frac{dy}{dx} = \cos x$

i.e $\frac{d}{dx}(\sin x) = \cos x.$

(5) cos x

Let $y = \cos x$ —(1)

Giving small increments Δx + Δy to x + y resp, we get

$y + \Delta y = \cos(x+\Delta x)$ —(2)

subtracting (1) from (2), we get

$y + \Delta y - y = \cos(x+\Delta x) - \cos x$

or $\Delta y = \cos(x+\Delta x) - \cos x$ —(3)

using the formula

$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right) \times \sin\left(\frac{A-B}{2}\right)$

$\therefore \Delta y = -2\sin\left(\frac{x+\Delta x+x}{2}\right)\times\sin\left(\frac{\cancel{x}+\Delta x-\cancel{x}}{2}\right)$

or $\Delta y = -2\sin\left(\frac{2x+\Delta x}{2}\right)\sin\frac{\Delta x}{2}$

Dividing b/s by Δx, we get

$\frac{\Delta y}{\Delta x} = \frac{-2\sin\left(x+\frac{\Delta x}{2}\right)\sin\frac{\Delta x}{2}}{\Delta x}$

$= \frac{-\cancel{2}\sin\left(x+\frac{\Delta x}{2}\right)\sin\Delta x/2}{\cancel{2}\,\Delta x/2}$

or $\frac{\Delta y}{\Delta x} = -\sin\left(x+\frac{\Delta x}{2}\right)\frac{\sin\frac{\Delta x}{2}}{\Delta x/2}$

Taking $\lim_{\Delta x\to 0}$ to b/s, we get

$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0} -\sin\left(x+\frac{\Delta x}{2}\right)\times\frac{\sin\Delta x/2}{\Delta x/2}$

or $\frac{dy}{dx} = \lim_{\Delta x\to 0} -\sin\left(x+\frac{\Delta x}{2}\right)\times\lim_{\Delta x\to 0}\frac{\sin\frac{\Delta x}{2}}{\Delta x/2}$

or $\frac{dy}{dx} = -\sin(x+0)\times 1$ $\left(\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right)$

or $\frac{dy}{dx} = -\sin x$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\cos x) = -\sin x$