Methods of finding Derivative :- There are two methods of finding the derivative of a function, viz:
1. Abinitio method or 1st Principle.
2. Direct method.

(1) Abinitio method or 1st Principle :-
This method is given as follows:
let y be a function of x i.e

$$y = f(x) \quad \text{——} (1)$$

Since x is independent variable and y is dependent variable. Therefore by changing the value of x, the value of y also changes.

step 1. let x and y be increased by a small amount Δx and Δy respectively, called increment in x and y, then

$$y + \Delta y = f(x + \Delta x) \quad \text{——} (2)$$

step 2. Subtracting $=^n$ (1) from (2), we get

$$\cancel{y} + \Delta y - \cancel{y} = f(x + \Delta x) - f(x)$$

$$\Delta y = f(x + \Delta x) - f(x)$$

step 3. Dividing b/s by Δx, we get

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

step 4. taking limit $_{\Delta x \to 0}$ (i.e $\lim\limits_{\Delta x \to 0}$) to b/s, so as to obtain instantaneous rate of change of y w.r.t x

$$\therefore \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

(Conti)

The quantity $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ is represented by $\frac{dy}{dx}$ and is called differential coefficient or derivative or differentiation of y w.r.t. x.

Hence, $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$.

Here the ratio $\frac{dy}{dx}$ is called as the derivative or differential coefficient or differentiation of y w.r.t the independent variable x.

Example :- Differentiate ① x^n by abinitio method or Ist Principle.

$$\text{let } y = x^n \quad \text{—①}$$

Let Δx and Δy be the small increments in the value of x and y respectively, then from eqn ①, we have.

$$y + \Delta y = (x + \Delta x)^n \quad \text{—②}$$

subtracting eqn ① from eqn ②, we get

$$\cancel{y} + \Delta y - \cancel{y} = (x + \Delta x)^n - x^n$$

or $\Delta y = (x + \Delta x)^n - x^n$

or $\Delta y = x^n \left[1 + \frac{\Delta x}{x}\right]^n - x^n$

or $\Delta y = x^n \left\{\left(1 + \frac{\Delta x}{x}\right)^n - 1\right\} \quad \text{—③}$

using Binomial theorem in eqn ③ i.e

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \ldots$$

∴ eqn ③ becomes.

$$\therefore \Delta y = x^n \left[\cancel{1} \cancel{+} n \frac{\Delta x}{x} + \frac{n(n-1)}{2!} \left(\frac{\Delta x}{x}\right)^2 + \ldots \cancel{-1}\right]$$

(cont.)

or $\Delta y = x^n \left[n\frac{\Delta x}{x} + \frac{n(n-1)}{2!}\left(\frac{\Delta x}{x}\right)^2 + \text{———} \right]$

Dividing b/s by Δx, we have

$$\frac{\Delta y}{\Delta x} = \frac{x^n}{\Delta x}\left[n\frac{\Delta x}{x} + \frac{n(n-1)}{2!}\left(\frac{\Delta x}{x}\right)^2 + \text{———} \right]$$

or $$\frac{\Delta y}{\Delta x} = x^n \left[\frac{n}{x}\cdot\frac{\Delta x}{\Delta x} + \frac{n(n-1)}{2!}\frac{(\Delta x)^2}{\Delta x \cdot x^2} + \text{———} \right]$$

or $$\frac{\Delta y}{\Delta x} = x^n \left[\frac{n}{x} + \frac{n(n-1)}{2!}\frac{\Delta x}{x^2} + \text{———} \right]$$

Taking $\lim_{\Delta x \to 0}$ to b/s we get

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} x^n \left[\frac{n}{x} + \frac{n(n-1)}{2!}\frac{\Delta x}{x^2} + \text{———} \right]$$

or $$\frac{dy}{dx} = \frac{nx^n}{x} + 0 = nx^n \cdot x^{-1}$$

or $$\boxed{\frac{dy}{dx} = nx^{n-1}} \quad \text{——} (4)$$

using eqn (4) in eqn (1), we get

$$\boxed{\frac{d(x)^n}{dx} = nx^{n-1}}$$

(2) $(ax+b)^n \rightarrow$ Differentiate by abinitio method

Sol:- Let $y = (ax+b)^n$ —— (1)

Let Δx and Δy be increments in the value of x and y respectively, then

$$y + \Delta y = [a(x+\Delta x)+b]^n \quad \text{——} (2)$$

subtrating eqn (1) from eqn (2);

$$\therefore y + \Delta y - y = [a(x+\Delta x)+b]^n - (ax+b)^n$$

$$\Delta y = [a(x+\Delta x)+b]^n - (ax+b)^n$$