SETS

Exercise 1.3

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Question 1:

Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces:

(i) {2, 3, 4} … {1, 2, 3, 4, 5}

(ii) {*a*, *b*, *c*} … {*b*, *c*, *d*}

(iii) {*x*: *x* is a student of Class XI of your school} … {*x*: *x* student of your school}

(iv) {*x*: *x* is a circle in the plane} … {*x*: *x* is a circle in the same plane with radius 1 unit}

(v) {*x*: *x* is a triangle in a plane}…{*x*: *x* is a rectangle in the plane}

(vi) {*x*: *x* is an equilateral triangle in a plane}… {*x*: *x* is a triangle in the same plane}

(vii) {*x*: *x* is an even natural number} … {*x*: *x* is an integer}

Answer 1:

(i) $\{2,3,4\} \subset \{1,2,3,4,5\}$

(ii) $\{a,b,c\} \not\subset \{b,c,d\}$

(iii) {*x*: *x* is a student of class XI of your school}$\subset$ {*x*: *x* is student of your school}

(iv) {*x*: *x* is a circle in the plane} $\not\subset$ {*x*: *x* is a circle in the same plane with radius 1 unit}

(v) {*x*: *x* is a triangle in a plane} $\not\subset$ {*x*: *x* is a rectangle in the plane}

(vi) {*x*: *x* is an equilateral triangle in a plane}$\subset$ {*x*: *x* in a triangle in the same plane} (vii) {*x*: *x* is an even natural number} $\subset$ {*x*: *x* is an integer}

Question 2:

Examine whether the following statements are true or false:

(i) {*a*, *b*} $\not\subset$ {*b*, *c*, *a*}

(ii) {*a*, *e*} $\subset$ {*x*: *x* is a vowel in the English alphabet}

(iii) {1, 2, 3} $\subset${1, 3, 5}

(iv) {*a*} $\subset$ {*a*. *b*, *c*}

(v) {*a*} $\in$ (*a*, *b*, *c*)

(vi) {*x*: *x* is an even natural number less than 6} $\subset$ {*x*: *x* is a natural number which divides 36}

Answer 2:

(i) False. Each element of {*a*, *b*} is also an element of {*b*, *c*, *a*}.

(ii) True. *a*, *e* are two vowels of the English alphabet.

(iii) False. $2 \in \{1, 2, 3\}$; however, $2 \notin \{1, 3, 5\}$

(iv) True. Each element of $\{a\}$ is also an element of $\{a, b, c\}$.

(v) False. The elements of $\{a, b, c\}$ are a, b, c. Therefore, $\{a\} \subset \{a, b, c\}$

(vi) True. $\{x : x$ is an even natural number less than $6\} = \{2, 4\}$

$\{x : x$ is a natural number which divides $36\} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

Question 3:

Let A= {1, 2, {3, 4,}, 5}. Which of the following statements are incorrect and why?

(i) $\{3, 4\} \subset A$

(ii) $\{3, 4\}\} \in A$

(iii) $\{\{3, 4\}\} \subset A$

(iv) $1 \in A$

(v) $1 \subset A$

(vi) $\{1, 2, 5\} \subset A$

(vii) $\{1, 2, 5\} \in A$

(viii) $\{1, 2, 3\} \subset A$

(ix) $\Phi \in A$

(x) $\Phi \subset A$

(xi) $\{\Phi\} \subset A$

Answer 3:

$A = \{1, 2, \{3, 4\}, 5\}$

(i) The statement $\{3, 4\} \subset A$ is incorrect because $3 \in \{3, 4\}$; however, $3 \notin A$.

(ii) The statement $\{3, 4\} \in A$ is correct because $\{3, 4\}$ is an element of A.

(iii) The statement $\{\{3, 4\}\} \subset A$ is correct because $\{3, 4\} \in \{\{3, 4\}\}$ and $\{3, 4\} \in A$.

(iv) The statement $1 \in A$ is correct because 1 is an element of A.

(v) The statement $1 \subset A$ is incorrect because an element of a set can never be a subset of itself.

(vi) The statement $\{1, 2, 5\} \subset A$ is correct because each element of $\{1, 2, 5\}$ is also an element of A.

(vii) The statement $\{1, 2, 5\} \in A$ is incorrect because $\{1, 2, 5\}$ is not an element of A.

(viii) The statement $\{1, 2, 3\} \subset A$ is incorrect because $3 \in \{1, 2, 3\}$; however, $3 \notin A$.

(ix) The statement $\Phi \in A$ is incorrect because Φ is not an element of A.

(x) The statement $\Phi \subset A$ is correct because Φ is a subset of every set.

(xi) The statement $\{\Phi\} \subset A$ is incorrect because $\Phi \in \{\Phi\}$; however, $\Phi \in A$.

Question 4:

Write down all the subsets of the following sets:

(i) $\{a\}$

(ii) $\{a, b\}$

(iii) $\{1, 2, 3\}$

(iv) Φ

Answer 4:

(i) The subsets of $\{a\}$ are Φ and $\{a\}$.

(ii) The subsets of $\{a, b\}$ are Φ, $\{a\}$, $\{b\}$, and $\{a, b\}$.

(iii) The subsets of $\{1, 2, 3\}$ are Φ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{2, 3\}$, $\{1, 3\}$ and $\{1, 2, 3\}$

(iv) The only subset of Φ is Φ.

Question 5:

How many elements has P(A), if A = Φ?

Answer 5:

We know that if A is a set with m elements i.e., $n(A) = m$, then $n[P(A)] = 2^m$.

If $A = \Phi$, then $n(A) = 0$.

$\therefore \quad n[P(A)] = 2^0 = 1$

Hence, P(A) has one element.

Question 6:

Write the following as intervals:

(i) $\{x: x \in R, -4 < x \leq 6\}$

(ii) $\{x: x \in R, -12 < x < -10\}$

(iii) $\{x: x \in R, 0 \leq x < 7\}$

(iv) $\{x: x \in R, 3 \leq x \leq 4\}$

Answer 6:

(i) $\{x: x \in R, -4 < x \leq 6\} = (-4, 6]$

(ii) $\{x: x \in R, -12 < x < -10\} = (-12, -10)$

(iii) $\{x: x \in R, 0 \leq x < 7\} = [0, 7)$

(iv) $\{x: x \in R, 3 \leq x \leq 4\} = [3, 4]$

Question 7:

Write the following intervals in set-builder form:

(i) (–3, 0)

(ii) [6, 12]

(iii) (6, 12]

(iv) [–23, 5)

Answer 7:

(i) $(-3, 0) = \{x: x \in R, -3 < x < 0\}$

(ii) $[6, 12] = \{x: x \in R, 6 \le x \le 12\}$

(iii) $(6, 12] = \{x: x \in R, 6 < x \le 12\}$

(iv) $[-23, 5) = \{x: x \in R, -23 \le x < 5\}$

Question 8:

What universal set (s) would you propose for each of the following?

(i) The set of right triangles

(ii) The set of isosceles triangles

Answer 8:

(i) For the set of right triangles, the universal set can be the set of triangles or the set of polygons.

(ii) For the set of isosceles triangles, the universal set can be the set of triangles or the set of polygons or the set of two-dimensional figures.

Question 9:

Given the sets A = {1, 3, 5}, B = {2, 4, 6} and C = {0, 2, 4, 6, 8}, which of the following may be considered as universals set (s) for all the three sets A, B and C

(i) {0, 1, 2, 3, 4, 5, 6}

(ii) Φ

(iii) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(iv) {1, 2, 3, 4, 5, 6, 7, 8}

Answer 9:

(i) It can be seen that A ⊂ {0, 1, 2, 3, 4, 5, 6}

B ⊂ {0, 1, 2, 3, 4, 5, 6}

However, C ⊄ {0, 1, 2, 3, 4, 5, 6}

Therefore, the set {0, 1, 2, 3, 4, 5, 6} cannot be the universal set for the sets A, B, and C.

(ii) A ⊄ Φ, B ⊄ Φ, C ⊄ Φ

Therefore, Φ cannot be the universal set for the sets A, B, and C.

(iii) A ⊂ {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

B ⊂ {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

C ⊂ {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Therefore, the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} is the universal set for the sets A, B, and C.

(iv) A ⊂ {1, 2, 3, 4, 5, 6, 7, 8}

B ⊂ {1, 2, 3, 4, 5, 6, 7, 8}

However, C ⊄ {1, 2, 3, 4, 5, 6, 7, 8}

Therefore, the set {1, 2, 3, 4, 5, 6, 7, 8} cannot be the universal set for the sets A, B, and C.

Exercise 1.4

Question 1:

Find the union of each of the following pairs of sets:

(i) X = {1, 3, 5}; Y = {1, 2, 3}

(ii) A = {a, e, i, o, u}; B = {a, b, c}

(iii) A = {x: x is a natural number and multiple of 3}

B = {x: x is a natural number less than 6}

(iv) A = {x: x is a natural number and $1 < x \le 6$}

B = {x: x is a natural number and $6 < x < 10$}

(v) A = {1, 2, 3}; B = Φ

Answer 1:

(i) X = {1, 3, 5} Y = {1, 2, 3}

X ∪ Y= {1, 2, 3, 5}

(ii) A = {a, e, i, o, u} B = {a, b, c}

A ∪ B = {a, b, c, e, i, o, u}

(iii) A = {x: x is a natural number and multiple of 3} = {3, 6, 9 …}

B = {x: x is a natural number less than 6} = {1, 2, 3, 4, 5, 6}

A ∪ B = {1, 2, 4, 5, 3, 6, 9, 12 …}

∴ A ∪ B = {x: x = 1, 2, 4, 5 or a multiple of 3}

(iv) A = {x: x is a natural number and $1 < x \le 6$} = {2, 3, 4, 5, 6}

B = {x: x is a natural number and $6 < x < 10$} = {7, 8, 9}

A ∪ B = {2, 3, 4, 5, 6, 7, 8, 9}

∴ A∪ B = {x: $x \in$ N and $1 < x < 10$}

(v) A = {1, 2, 3}, B = Φ

A ∪ B = {1, 2, 3}

Question 2:

Let A = {a, b}, B = {a, b, c}. Is A ⊂ B? What is A ∪ B?

Answer 2:

Here, A = {a, b} and B = {a, b, c}

Yes, A ⊂ B.

A ∪ B = {a, b, c} = B

Question 3:

If A and B are two sets such that A ⊂ B, then what is A ∪ B?

Answer 3:

If A and B are two sets such that A ⊂ B, then A ∪ B = B.

Question 4:

If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {5, 6, 7, 8} and D = {7, 8, 9, 10}; find

(i) A ∪ B

(ii) A UC

(iii) B ∪ C

(iv) B ∪ D

(v) A ∪ B ∪ C

(vi) A ∪ B ∪ D

(vii) B ∪ C ∪ D

Answer 5:

A = {1, 2, 3, 4], B = {3, 4, 5, 6}, C = {5, 6, 7, 8} and D = {7, 8, 9, 10}

(i) A ∪ B = {1, 2, 3, 4, 5, 6}

(ii) A ∪ C = {1, 2, 3, 4, 5, 6, 7, 8}

(iii) B ∪ C = {3, 4, 5, 6, 7, 8}

(iv) B ∪ D = {3, 4, 5, 6, 7, 8, 9, 10}

(v) A ∪ B ∪ C = {1, 2, 3, 4, 5, 6, 7, 8}

(vi) A ∪ B ∪ D = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(vii) B ∪ C ∪ D = {3, 4, 5, 6, 7, 8, 9, 10}

Question 5:

Find the intersection of each pair of sets:

(i) X = {1, 3, 5} Y = {1, 2, 3}

(ii) A = {*a*, *e*, *i*, *o*, *u*} B = {*a*, *b*, *c*}

(iii) A = {*x*: *x* is a natural number and multiple of 3}

B = {*x*: *x* is a natural number less than 6}

(iv) A = {*x*: *x* is a natural number and $1 < x \leq 6$}

B = {*x*: *x* is a natural number and $6 < x < 10$}

(v) A = {1, 2, 3}, B = Φ

Answer 5:

(i) X = {1, 3, 5}, Y = {1, 2, 3}

X ∩ Y = {1, 3}

(ii) A = {*a*, *e*, *i*, *o*, *u*}, B = {*a*, *b*, *c*}

A ∩ B = {*a*}

(iii) A = {*x*: *x* is a natural number and multiple of 3} = (3, 6, 9 …}

B = {*x*: *x* is a natural number less than 6} = {1, 2, 3, 4, 5}

∴ A ∩ B = {3}

(iv) A = {*x*: *x* is a natural number and $1 < x \leq 6$} = {2, 3, 4, 5, 6}

B = {x: x is a natural number less than 6} = {1, 2, 3, 4, 5}

∴ A ∩ B = {3}

(iv) A = {x: x is a natural number and $1 < x \le 6$} = {2, 3, 4, 5, 6}

B = {x: x is a natural number and $6 < x < 10$}

= {7, 8, 9}

A ∩ B = Φ

(v) A = {1, 2, 3}, B = Φ. So, A ∩ B = Φ

Question 6:

If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13}, C = {11, 13, 15} and D = {15, 17}; find

(i) A ∩ B

(ii) B ∩ C

(iii) A ∩ C ∩ D

(iv) A ∩ C

(v) B ∩ D

(vi) A ∩ (B ∪ C)

(vii) A ∩ D

(viii) A ∩ (B ∪ D)

(ix) (A ∩ B) ∩ (B ∪ C)

(x) (A ∪ D) ∩ (B ∪ C)

Answer 6:

(i) A ∩ B = {7, 9, 11}

(ii) B ∩ C = {11, 13}

(iii) A ∩ C ∩ D = { A ∩ C} ∩ D = {11} ∩ {15, 17} = Φ

(iv) A ∩ C = {11}

(v) B ∩ D = Φ

(vi) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

= {7, 9, 11} ∪ {11} = {7, 9, 11}

(vii) A ∩ D = Φ

(viii) A ∩ (B ∪ D) = (A ∩ B) ∪ (A ∩ D)

= {7, 9, 11} ∪ Φ = {7, 9, 11}

(ix) (A ∩ B) ∩ (B ∪ C) = {7, 9, 11} ∩ {7, 9, 11, 13, 15} = {7, 9, 11}

(x) (A ∪ D) ∩ (B ∪ C) = {3, 5, 7, 9, 11, 15, 17) ∩ {7, 9, 11, 13, 15}

= {7, 9, 11, 15}

Question 7:

If A = {x: x is a natural number}, B ={x: x is an even natural number}

C = {x: x is an odd natural number} and D = {x: x is a prime number},

find

(i) A ∩ B

(ii) A ∩ C

(iii) A ∩ D

(iv) B ∩ C

(v) B ∩ D

(vi) C ∩ D

Answer 7:

A = {x: x is a natural number} = {1, 2, 3, 4, 5 …}

B ={x: x is an even natural number} = {2, 4, 6, 8 …}

C = {x: x is an odd natural number} = {1, 3, 5, 7, 9 …}

D = {x: x is a prime number} = {2, 3, 5, 7 …}

(i) A ∩B = {x: x is a even natural number} = B

(ii) A ∩ C = {x: x is an odd natural number} = C

(iii) A ∩ D = {x: x is a prime number} = D

(iv) B ∩ C = Φ

(v) B ∩ D = {2}

(vi) C ∩ D = {x: x is odd prime number}

Question 8:

Which of the following pairs of sets are disjoint

(i) {1, 2, 3, 4} and {x: x is a natural number and $4 \le x \le 6$}

(ii) {a, e, i, o, u} and {c, d, e, f}

(iii) {x: x is an even integer} and {x: x is an odd integer}

Answer 8:

(i) {1, 2, 3, 4}

{x: x is a natural number and $4 \le x \le 6$} = {4, 5, 6}

Now, {1, 2, 3, 4} ∩ {4, 5, 6} = {4}

Therefore, this pair of sets is not disjoint.

(ii) {a, e, i, o, u} ∩ (c, d, e, f} = {e}

Therefore, {a, e, i, o, u} and (c, d, e, f} are not disjoint.

(iii) {x: x is an even integer} ∩ {x: x is an odd integer} = Φ

Therefore, this pair of sets is disjoint.

Question 9:

If A = {3, 6, 9, 12, 15, 18, 21}, B = {4, 8, 12, 16, 20},
C = {2, 4, 6, 8, 10, 12, 14, 16}, D = {5, 10, 15, 20}; find

(i) A – B

(ii) A – C

(iii) A – D

(iv) B – A

(v) C – A

(vi) D – A

(vii) B – C

(viii) B – D

(ix) C – B

(x) D – B

(xi) C – D

(xii) D – C

Answer 9:

(i) A – B = {3, 6, 9, 15, 18, 21}

(ii) A – C = {3, 9, 15, 18, 21}

(iii) A – D = {3, 6, 9, 12, 18, 21}

(iv) B – A = {4, 8, 16, 20}

(v) C – A = {2, 4, 8, 10, 14, 16}

(vi) D – A = {5, 10, 20}

(vii) B – C = {20}

(viii) B – D = {4, 8, 12, 16}

(ix) C – B = {2, 6, 10, 14}

(x) D – B = {5, 10, 15}

(xi) C – D = {2, 4, 6, 8, 12, 14, 16}

(xii) D – C = {5, 15, 20}

Question 10:

If X = {*a*, *b*, *c*, *d*} and Y = {*f*, *b*, *d*, *g*}, find

(i) X – Y

(ii) Y – X

(iii) X ∩ Y

Answer 10:

(i) X − Y = {*a*, *c*}

(ii) Y − X = {*f*, *g*}

(iii) X ∩ Y = {*b*, *d*}

Question 11:

If R is the set of real numbers and Q is the set of rational numbers, then what is R − Q?

Answer 11:

R: set of real numbers

Q: set of rational numbers

Therefore, R − Q is a set of irrational numbers.

Question 12:

State whether each of the following statement is true or false. Justify your answer.

(i) {2, 3, 4, 5} and {3, 6} are disjoint sets.

(ii) {*a*, *e*, *i*, *o*, *u* } and {*a*, *b*, *c*, *d*} are disjoint sets.

(iii) {2, 6, 10, 14} and {3, 7, 11, 15} are disjoint sets.

(iv) {2, 6, 10} and {3, 7, 11} are disjoint sets.

Answer 12:

(i) False

As 3 ∈ {2, 3, 4, 5}, 3 ∈ {3, 6}

⇒ {2, 3, 4, 5} ∩ {3, 6} = {3}

(ii) False

As *a* ∈ {*a*, *e*, *i*, *o*, *u*}, *a* ∈ {*a*, *b*, *c*, *d*}

⇒ {*a*, *e*, *i*, *o*, *u* } ∩ {*a*, *b*, *c*, *d*} = {*a*}

(iii) True

As {2, 6, 10, 14} ∩ {3, 7, 11, 15} = Φ

(iv) True

As {2, 6, 10} ∩ {3, 7, 11} = Φ

Exercise 1.5

Question 1:

Let U ={1, 2, 3; 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6}. Find

(i) A'

(ii) B'

(iii) $(A \cup C)'$

(iv) $(A \cup B)'$

(v) $(A')'$

(vi) $(B - C)'$

Answer 1:

U ={1, 2, 3, 4, 5, 6, 7, 8, 9} A = {1, 2, 3, 4} B = {2, 4, 6, 8}
C = {3, 4, 5, 6}

(i) $A' = \{5, 6, 7, 8, 9\}$

(ii) $B' = \{1, 3, 5, 7, 9\}$

(iii) $A \cup C = \{1, 2, 3, 4, 5, 6\}$

$\therefore (A \cup C)' = \{7, 8, 9\}$

(iv) $A \cup B = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B)' = \{5, 7, 9\}$

(v) $(A')' = A = \{1, 2, 3, 4\}$

(vi) $B - C = \{2, 8\}$

$\therefore (B - C)' = \{1, 3, 4, 5, 6, 7, 9\}$

Question 2:

If U = {*a, b, c, d, e, f, g, h*}, find the complements of the following sets:

(i) A = {*a, b, c*}

(ii) B = {*d, e, f, g*}

(iii) C = {*a, c, e, g*}

(iv) D = {*f, g, h, a*}

Answer 2:

U = {*a, b, c, d, e, f, g, h*}

(i) A = {*a, b, c*}

$A' = \{d, e, f, g, h\}$

(ii) B = {*d, e, f, g*}

$\therefore B' = \{a, b, c, h\}$

(iii) C = {a, c, e, g}

$\therefore C' = \{b, d, f, h\}$

(iv) D = {f, g, h, a}

$\therefore D' = \{b, c, d, e\}$

Question 3:

Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) {x: x is an even natural number}

(ii) {x: x is an odd natural number}

(iii) {x: x is a positive multiple of 3}

(iv) {x: x is a prime number}

(v) {x: x is a natural number divisible by 3 and 5}

(vi) {x: x is a perfect square}

(vii) {x: x is perfect cube}

(viii) {x: $x + 5 = 8$} (ix) {x: $2x + 5 = 9$}

(x) {x: $x \geq 7$}

(xi) {x: $x \in$ N and $2x + 1 > 10$}

Answer 3:

U = N: Set of natural numbers

(i) {x: x is an even natural number}´ = {x: x is an odd natural number}

(ii) {x: x is an odd natural number}´ = {x: x is an even natural number}

(iii) {x: x is a positive multiple of 3}´= {x: $x \in$ N and x is not a multiple of 3}

(iv) {x: x is a prime number}´ ={x: x is a positive composite number and x = 1}

(v) {x: x is a natural number divisible by 3 and 5}´ = {x: x is a natural number that is not divisible by 3 or 5}

(vi) {x: x is a perfect square}´ = {x: $x \in$ N and x is not a perfect square}

(vii) {x: x is a perfect cube}´ = {x: $x \in$ N and x is not a perfect cube}

(viii) {x: $x + 5 = 8$}´ = {x: $x \in$ N and $x \neq 3$}

(ix) $\{x: 2x + 5 = 9\}' = \{x: x \in N \text{ and } x \neq 2\}$

(x) $\{x: x \geq 7\}' = \{x: x \in N \text{ and } x < 7\}$

(xi) $\{x: x \in N \text{ and } 2x + 1 > 10\}' = \{x: x \in N \text{ and } x \leq 9/2\}$

Question 4:

If U = {1, 2, 3, 4, 5,6,7,8, 9},

A = {2, 4, 6, 8} and B = {2, 3, 5, 7}.

Verify that

(i) $(A \cup B)' = A' \cap B'$ **(ii)** $(A \cap B)' = A' \cup B'$

Answer 4:

U = {1, 2, 3, 4, 5, 6, 7, 8, 9}

A = {2, 4, 6, 8}, B = {2, 3, 5, 7}

(i)

$$(A \cup B)' = \{2, 3, 4, 5, 6, 7, 8\}' = \{1, 9\}$$
$$A' \cap B' = \{1, 3, 5, 7, 9\} \cap (1, 4, 6, 8, 9) = \{1, 9\}$$
$$\therefore (A \cup B)' = A' \cap B'$$

(ii)

$$(A \cap B)' = \{2\}' = \{1, 3, 4, 5, 6, 7, 8, 9\}$$
$$A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\} = \{1, 3, 4, 5, 6, 7, 8, 9\}$$
$$\therefore (A \cap B)' = A' \cup B'$$

Question 5:

Draw appropriate Venn diagram for each of the following:

(i) $(A \cup B)'$

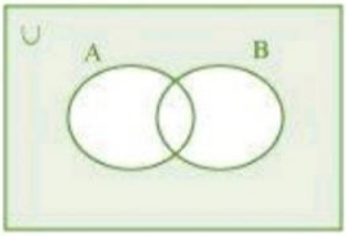
(ii) $A' \cap B'$

(iii) $(A \cap B)'$

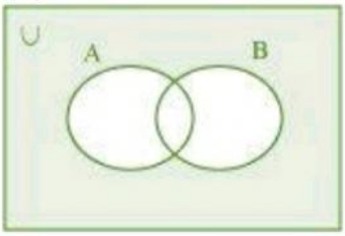
(iv) $A' \cup B'$

Answer 5:

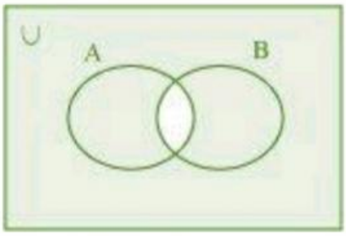
(i) $(A \cup B)'$



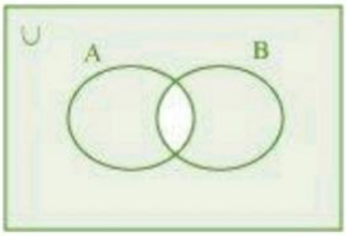
(ii) $A' \cap B'$



(iii) $(A \cap B)'$



(iv) $A' \cup B'$



Question 6:

Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60°, what is A'?

Answer 6:

A' is the set of all equilateral triangles.

Question 7:

Fill in the blanks to make each of the following a true statement:

(i) $A \cup A' = \ldots$

(ii) $\Phi' \cap A = \ldots$

(iii) $A \cap A' = \ldots$

(iv) $U' \cap A = \ldots$

Answer 7:

(i) $A \cup A' = U$

(ii) $\Phi' \cap A = U \cap A = A$

$\therefore \Phi' \cap A = A$

(iii) $A \cap A' = \Phi$

(iv) $U' \cap A = \Phi \cap A = \Phi$

$\therefore U' \cap A = \Phi$

Exercise 1.6

Question 1:

If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$.

Answer 1:

It is given that:

$n(X) = 17$, $n(Y) = 23$, $n(X \cup Y) = 38$

$n(X \cap Y) = ?$

We know that:

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$
$$\therefore 38 = 17 + 23 - n(X \cap Y)$$
$$\Rightarrow n(X \cap Y) = 40 - 38 = 2$$
$$\therefore n(X \cap Y) = 2$$

Question 2:

If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements; how many elements does $X \cap Y$ have?

Answer 2:

It is given that:

$$n(X \cup Y) = 18,\ n(X) = 8,\ n(Y) = 15$$

n(X ïf? Y) = ?

We know that:

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$
$$\therefore 18 = 8 + 15 - n(X \cap Y)$$
$$\Rightarrow n(X \cap Y) = 23 - 18 = 5$$
$$\therefore n(X \cap Y) = 5$$

Question 3:

In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Answer 3:

Let H be the set of people who speak Hindi, and E be the set of people who speak English

$\therefore n(H \cup E) = 400$, $n(H) = 250$, $n(E) = 200$ $\quad n(H \cap E) = ?$

We know that: $n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$\therefore 400 = 250 + 200 - n(H \cap E)$

$\Rightarrow 400 = 450 - n(H \cap E) \Rightarrow n(H \cap E) = 450 - 400$

$\therefore n(H \cap E) = 50$

Thus, 50 people can speak both Hindi and English.

Question 4:

If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?

Answer 4:

It is given that: $n(S) = 21$, $n(T) = 32$, $n(S \cap T) = 11$

We know that: $n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$\therefore n(S \cup T) = 21 + 32 - 11 = 42$

Thus, the set $(S \cup T)$ has 42 elements.

Question 5:

If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

Answer 5:

It is given that: $n(X) = 40$, $n(X \cup Y)$

$= 60$, $n(X \cap Y) = 10$ We know that:

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$ $\therefore$

$60 = 40 + n(Y) - 10$

$\therefore n(Y) = 60 - (40 - 10) = 30$

Thus, the set Y has 30 elements.

Question 6:

In a group of 70 people, 37 like coffee, 52 like tea, and each person likes at least one of the two drinks. How many people like both coffee and tea?

Answer 6:

Let C denote the set of people who like coffee, and T denote the set of people who like tea $n(C \cup T) = 70$, $n(C) = 37$, $n(T) = 52$ We know that:

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$ $\therefore 70 = 37 + 52 - n(C \cap T)$

$\Rightarrow 70 = 89 - n(C \cap T)$

$\Rightarrow n(C \cap T) = 89 - 70 = 19$

Thus, 19 people like both coffee and tea.

Question 7:

In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Answer 7:

Let C denote the set of people who like cricket, and T denote the set of people who like tennis

$\therefore n(C \cup T) = 65, \ n(C) = 40, \ n(C \cap T) = 10$

We know that: $n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$\therefore 65 = 40 + n(T) - 10$

$\Rightarrow 65 = 30 + n(T)$

$\Rightarrow n(T) = 65 - 30 = 35$

Therefore, 35 people like tennis.

Now,

$(T - C) \cup (T \cap C) = T$

Also,

$(T - C) \cap (T \cap C) = \Phi$

$\therefore n\ (T) = n\ (T - C) + n\ (T \cap C)$

$\Rightarrow 35 = n\ (T - C) + 10$

$\Rightarrow n\ (T - C) = 35 - 10 = 25$

Thus, 25 people like only tennis.

Question 8:

In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Answer 8:

Let F be the set of people in the committee who speak French, and

S be the set of people in the committee who speak Spanish

$\therefore n(F) = 50, \qquad n(S) = 20, \qquad n(S \cap F) = 10$

We know that: $n(S \cup F) = n(S) + n(F) - n(S \cap F)$

$= 20 + 50 - 10$

$= 70 - 10 = 60$

Thus, 60 people in the committee speak at least one of the two languages.