CHAPTER-1 SETS

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SET

A set is a well-defined collection of distinct objects. Well-defined collection means that there exists a rule with the help of which it is possible to tell whether a given object belongs or does not belong to given collection. Generally sets are denoted by capital letters A, B, C, X, Y, Z etc.

Types of Sets

Let us discuss here, types of sets:

Empty Sets

The set, which has no elements or null elements. This is also called a Null set or Void set. It is denoted by {}.

For example: Let, Set X = {x:x is the number of students studying in Class 6th and Class 7th}

Since we know a student cannot learn simultaneously on two classes, therefore set X is an empty set.

Another example is, set Y = {a: 1<a<2, a is a natural number}, we know natural cannot be a decimal, therefore set Y is a null set or empty set.

Singleton Sets

The set which has only one element is called a singleton set.

For example, Set X = { 2 } is a singleton set.

Finite and Infinite Sets

Finite sets are the one which has a finite number of elements and Infinite sets are those whose number of elements cannot be estimated but it has some figure or number, which is very large to express in a set.

For example, Set X = {1,2,3,4,5} is a finite set, as it has a finite number of elements in it.

Set Y = {Number of Animals in India} is an infinite set, as there is an approximate number of Animals in India, but actual value cannot be expressed, as the numbers could be very large.

Equal Sets

Two sets X and Y are said to be equal if every element of set X is also the elements of set Y and if every element of set Y is also the elements of set X. It means set X and set Y have the same elements and we can denote it as;

$X = Y$

For example, Let $X = \{ 1,2,3,4\}$ and $Y = \{4,3,2,1\}$, then $X = Y$

And if X = {set of even numbers} and Y = { set of natural numbers} the $X \neq Y$, because natural numbers consist of all the positive integers starting from 1,2,3,4,5 to infinity, but even numbers starts with 2,4,6,8, and so on.

Subsets

A set X is said to be a subset of set Y if the elements of set X belongs to set Y or you can say each element of set X is present in set Y. It is denoted with the symbol as $X \subset Y$.

We can also write the subset notation as;

$X \subset Y$ if $a \in X \Rightarrow a \in Y$

Thus, from the above equation, “X is a subset of Y if a is an element of X implies that a is also an element of Y”.

Each set is a subset of its own set and a null set or empty set is a subset of all sets.

Power Sets

The power set is nothing but the set of all subsets. Let us explain how.

We know the empty set is a subset of all sets and every set is a subset of itself. Taking an example of set X = {2,3}. From the above given statements we can write,

{} is a subset of {2,3}

{2} is a subset of {2,3}

{3} is a subset of {2,3}

{2,3} is also a subset of {2,3}

Therefore, power set of X = {2,3},

P(X) = {{},{2},{3},{2,3}}

Universal Sets

A universal set is a set which contains all the elements of other sets. Generally, it is represented as 'U'.

For example; set X = {1,2,3}, set Y = {3,4,5,6} and Z = {5,6,7,8,9}

Then, we can write universal set as, U = {1,2,3,4,5,6,7,8,9,}

Note: From the definition of the universal set, we can say, all the sets are subsets of the universal set. Therefore,

$X \subset U$

$Y \subset U$

And $Z \subset U$

Union Of sets

A union of two sets has all their elements. It is denoted by $\cup$.

For example, set X = {2,3,7} and set Y = { 4,5,8}

Then union of set X and set Y will be;

$X \cup Y = \{2,3,7,4,5,8\}$

Properties of Union of sets:

$X \cup Y = Y \cup X$; Commutative law

$(X \cup Y) \cup Z = X \cup (Y \cup Z)$

$X \cup \{\} = X$; $\{\}$ is the identity of $\cup$

$X \cup X = X$

$U \cup X = U$

Intersection of sets

Set of all elements, which are common to all the given sets, gives intersection of sets. It is denoted by ∩.

For example, set X = {2,3,7} and set Y = {2,4,9}

So, X ∩ Y = {2}

Difference of sets

The difference of set X and set Y is such that, it has only those elements which are in the set X and not in the set Y.

i.e. $X - Y = \{a: a \in X \text{ and } a \notin Y\}$

In the same manner, $Y - X = \{a: a \in Y \text{ and } a \notin X\}$

For example, if set X = {a, b, c, d} and Y = {b, c, e, f} then,

X − Y = {a, d} and Y − X = {e, f}

Disjoint Sets

If two sets X and Y have no common elements and their intersection results in zero(0), then set X and Y are called disjoint sets.

It can be represented as; $X \cap Y = 0$

Exercise 1.1

Question 1:

Which of the following are sets? Justify your answer.

(i) The collection of all months of a year beginning with the letter J.

(ii) The collection of ten most talented writers of India.

(iii) A team of eleven best-cricket batsmen of the world.

(iv) The collection of all boys in your class.

(v) The collection of all natural numbers less than 100.

(vi) A collection of novels written by the writer Munshi Prem Chand.

(vii) The collection of all even integers.

(viii) The collection of questions in this Chapter.

(ix) A collection of most dangerous animals of the world.

Answer 1:

(i) The collection of all months of a year beginning with the letter J is a well-defined collection of objects because one can definitely identify a month that belongs to this collection.

Hence, this collection is a set.

(ii) The collection of ten most talented writers of India is not a well-defined collection because the criteria for determining a writer's talent may vary from person to person.

Hence, this collection is not a set.

(iii) A team of eleven best cricket batsmen of the world is not a well-defined collection because the criteria for determining a batsman's talent may vary from person to person.

Hence, this collection is not a set.

(iv) The collection of all boys in your class is a well-defined collection because you can definitely identify a boy who belongs to this collection.

Hence, this collection is a set.

(v) The collection of all natural numbers less than 100 is a well-defined collection because one can definitely identify a number that belongs to this collection.

Hence, this collection is a set.

(vi) A collection of novels written by the writer Munshi Prem Chand is a well-defined collection because one can definitely identify a book that belongs to this collection.

Hence, this collection is a set.

(vii) The collection of all even integers is a well-defined collection because one can definitely identify an even integer that belongs to this collection.
Hence, this collection is a set.

(viii) The collection of questions in this chapter is a well-defined collection because one can definitely identify a question that belongs to this chapter.
Hence, this collection is a set.

(ix) The collection of most dangerous animals of the world is not a well-defined collection because the criteria for determining the dangerousness of an animal can vary from person to person.
Hence, this collection is not a set.

Question 2:

Let A = {1, 2, 3, 4, 5, 6}. Insert the appropriate symbol $\in$or $\notin$ in the blank spaces:

(i) 5...A **(ii)** 8...A **(iii)** 0...A

(iv) 4...A **(v)** 2...A **(vi)** 10...A

Answer 2:

(i) $5 \in A$

(ii) $8 \notin A$

(iii) $0 \notin A$

(iv) $4 \in A$

(v) $2 \in A$

(vi) $10 \notin A$

Question 3:

Write the following sets in roster form:

(i) A = $\{x: x$ is an integer and $-3 < x < 7\}$.

(ii) B = $\{x: x$ is a natural number less than 6$\}$.

(iii) C = $\{x: x$ is a two-digit natural number such that the sum of its digits is 8$\}$

(iv) D = {x: x is a prime number which is divisor of 60}.

(v) E = The set of all letters in the word TRIGONOMETRY.

(vi) F = The set of all letters in the word BETTER.

Answer 3:

(i) A = {x: x is an integer and $-3 < x < 7$}

The elements of this set are −2, −1, 0, 1, 2, 3, 4, 5, and 6 only.

Therefore, the given set can be written in roster form as

A = {−2, −1, 0, 1, 2, 3, 4, 5, 6}

(ii) B = {x: x is a natural number less than 6}

The elements of this set are 1, 2, 3, 4, and 5 only.

Therefore, the given set can be written in roster form as

B = {1, 2, 3, 4, 5}

(iii) C = {x: x is a two-digit natural number such that the sum of its digits is 8} The elements of this set are 17, 26, 35, 44, 53, 62, 71, and 80 only.

Therefore, this set can be written in roster form as

C = {17, 26, 35, 44, 53, 62, 71, 80}

(iv) D = {x: x is a prime number which is a divisor of 60}

$60 = 2 \times 2 \times 3 \times 5$

The elements of this set are 2, 3, and 5 only.

Therefore, this set can be written in roster form as D = {2, 3, 5}.

(v) E = The set of all letters in the word TRIGONOMETRY

There are 12 letters in the word TRIGONOMETRY, out of which letters T, R, and O are repeated.

Therefore, this set can be written in roster form as

E = {T, R, I, G, O, N, M, E, Y}

(vi) F = The set of all letters in the word BETTER

There are 6 letters in the word BETTER, out of which letters E and T are repeated.

Therefore, this set can be written in roster form as

F = {B, E, T, R}

Question 4:

Write the following sets in the set-builder form:

(i) (3, 6, 9, 12)

(ii) {2, 4, 8, 16, 32}

(iii) {5, 25, 125, 625}

(iv) {2, 4, 6 ...}

(v) {1, 4, 9 ... 100}

Answer 4:

(i) $\{3, 6, 9, 12\} = \{x: x = 3n, n \in N \text{ and } 1 \leq n \leq 4\}$

(ii) {2, 4, 8, 16, 32}

It can be seen that $2 = 2^1$, $4 = 2^2$, $8 = 2^3$, $16 = 2^4$, and $32 = 2^5$.

$\therefore \{2, 4, 8, 16, 32\} = \{x: x = 2^n, n \in N \text{ and } 1 \leq n \leq 5\}$

(iii) {5, 25, 125, 625}

It can be seen that $5 = 5^1$, $25 = 5^2$, $125 = 5^3$, and $625 = 5^4$.

$\therefore \{5, 25, 125, 625\} = \{x: x = 5^n, n \in N \text{ and } 1 \leq n \leq 4\}$

(iv) {2, 4, 6 ...}

It is a set of all even natural numbers.

$\therefore \{2, 4, 6 ...\} = \{x: x \text{ is an even natural number}\}$

(v) {1, 4, 9 ... 100}

It can be seen that $1 = 1^2$, $4 = 2^2$, $9 = 3^2$ ...$100 = 10^2$.

$\therefore \{1, 4, 9... 100\} = \{x: x = n^2, n \in N \text{ and } 1 \leq n \leq 10\}$

Question 5:

List all the elements of the following sets:

(i) A = {x: x is an odd natural number}

(ii) B = {x: x is an $-\frac{1}{2} < x < \frac{9}{2}$ integer,}

(iii) C = {x: x is an $x^2 \leq 4$} integer,

(iv) D = {x: x is a letter in the word "LOYAL"}

(v) E = {x: x is a month of a year not having 31 days}

(vi) F = {x: x is a consonant in the English alphabet which proceeds k}.

Answer 5:

(i) A = {x: x is an odd natural number} = {1, 3, 5, 7, 9 ...}

(ii) B = {x: x is an integer; $-\frac{1}{2} < n < \frac{9}{2}$}

It can be seen that $-\frac{1}{2} = -0.5$ and $\frac{9}{2} = 4.5$

$\therefore \quad = \{0,1,2,3,4\}$

(iii) C = {x: x is an integer; $x^2 \leq 4$}

It can be seen that

$(-1)^2 = 1 \leq 4$; $(-2)^2 = 4 \leq 4$; $(-3)^2 = 9 > 4$

$0^2 = 0 \leq 4$

$1^2 = 1 \leq 4$

$2^2 = 4 \leq 4$

$3^2 = 9 > 4$

$\therefore$C = {−2, −1, 0, 1, 2}

(iv) D = (x: x is a letter in the word "LOYAL") = {L, O, Y, A}

(v) E = {x: x is a month of a year not having 31 days}

= {February, April, June, September, November}

(vi) F = {x: x is a consonant in the English alphabet which precedes k}

= {b, c, d, f, g, h, j}

Question 6:

Match each of the set on the left in the roster form with the same set on the right described in set-builder form:

(I) {1, 2, 3, 6}	(A) {X: X IS A PRIME NUMBER AND A DIVISOR OF 6}
(II) {2, 3}	(B) {X: X IS AN ODD NATURAL NUMBER LESS THAN 10}
(III) {M, A,T, H, E, I,C, S}	(C) {X: X IS NATURAL NUMBER AND DIVISOR OF 6}
(IV) {1, 3, 5, 7, 9}	(D) {X: X IS A LETTER OF THE WORD MATHEMATICS}

Answer 6:

(i) All the elements of this set are natural numbers as well as the divisors of 6.

Therefore, (i) matches with (c).

(ii) It can be seen that 2 and 3 are prime numbers. They are also the divisors of 6.

(iii) All the elements of this set are letters of the word MATHEMATICS. Therefore, (iii) matches with (d).

(iv) All the elements of this set are odd natural numbers less than 10. Therefore, (iv) matches with (b).

Exercise 1.2

Question 1:

Which of the following are examples of the null set

(i) Set of odd natural numbers divisible by 2

(ii) Set of even prime numbers

(iii) $\{x:x$ is a natural numbers, $x < 5$ and $x > 7\}$

(iv) $\{y:y$ is a point common to any two parallel lines$\}$

Answer 1:

(i) A set of odd natural numbers divisible by 2 is a null set because no odd number is divisible by 2.

(ii) A set of even prime numbers is not a null set because 2 is an even prime number.

(iii) $\{x: x$ is a natural number, $x < 5$ and $x > 7\}$ is a null set because a number cannot be simultaneously less than 5 and greater than 7.

(iv) $\{y: y$ is a point common to any two parallel lines$\}$ is a null set because parallel lines do not intersect. Hence, they have no common point.

Question 2:

Which of the following sets are finite or infinite

(i) The set of months of a year

(ii) {1, 2, 3 ...}

(iii) {1, 2, 3 ... 99, 100}

(iv) The set of positive integers greater than 100

(v) The set of prime numbers less than 99

Answer 2:

(i) The set of months of a year is a finite set because it has 12 elements.

(ii) {1, 2, 3 ...} is an infinite set as it has infinite number of natural numbers.

(iii) {1, 2, 3 ...99, 100} is a finite set because the numbers from 1 to 100 are finite in number.

(iv) The set of positive integers greater than 100 is an infinite set because positive integers greater than 100 are infinite in number.

(v) The set of prime numbers less than 99 is a finite set because prime numbers less than 99 are finite in number.

Question 3:

State whether each of the following set is finite or infinite:

(i) The set of lines which are parallel to the x-axis

(ii) The set of letters in the English alphabet

(iii) The set of numbers which are multiple of 5

(iv) The set of animals living on the earth

(v) The set of circles passing through the origin (0, 0)

Answer 3:

(i) The set of lines which are parallel to the x-axis is an infinite set because lines parallel to the x-axis are infinite in number.

(ii) The set of letters in the English alphabet is a finite set because it has 26 elements.

(iii) The set of numbers which are multiple of 5 is an infinite set because multiples of 5 are infinite in number.

(iv) The set of animals living on the earth is a finite set because the number of animals living on the earth is finite (although it is quite a big number).

(v) The set of circles passing through the origin (0, 0) is an infinite set because infinite number of circles can pass through the origin.

Question 4:

In the following, state whether A = B or not:

(i) $A = \{a, b, c, d\}$; $B = \{d, c, b, a\}$

(ii) $A = \{4, 8, 12, 16\}$; $B = \{8, 4, 16, 18\}$

(iii) $A = \{2, 4, 6, 8, 10\}$; $B = \{x: x \text{ is positive even integer and } x \leq 10\}$

(iv) $A = \{x: x \text{ is a multiple of } 10\}$; $B = \{10, 15, 20, 25, 30 \ldots\}$

Answer 4:

(i) $A = \{a, b, c, d\}$; $B = \{d, c, b, a\}$

The order in which the elements of a set are listed is not significant.

$\therefore A = B$

(ii) $A = \{4, 8, 12, 16\}$; $B = \{8, 4, 16, 18\}$

It can be seen that $12 \in A$ but $12 \notin B$.

$\therefore A \neq B$

(iii) $A = \{2, 4, 6, 8, 10\}$

$B = \{x: x \text{ is a positive even integer and } x \leq 10\}$

$= \{2, 4, 6, 8, 10\}$

$\therefore A = B$

(iv) $A = \{x: x \text{ is a multiple of } 10\}$

$B = \{10, 15, 20, 25, 30 \ldots\}$

It can be seen that $15 \in B$ but $15 \notin A$.

$\therefore A \neq B$

Question 5:

Are the following pair of sets equal? Give reasons.

(i) $A = \{2, 3\}$; $B = \{x: x \text{ is solution of } x^2 + 5x + 6 = 0\}$

(ii) $A = \{x: x \text{ is a letter in the word FOLLOW}\}$; $B = \{y: y \text{ is a letter in the word WOLF}\}$

Answer 5:

(i) $A = \{2, 3\}$; $B = \{x: x \text{ is a solution of } x^2 + 5x + 6 = 0\}$

The equation $x^2 + 5x + 6 = 0$ can be solved as: $x(x + 3) + 2(x + 3) = 0$

$(x + 2)(x + 3) = 0$; $x = -2$ or $x = -3$

$\therefore$ $A = \{2, 3\}$; $B = \{-2, -3\}$

$\therefore$ $A \neq B$

(ii) $A = \{x: x \text{ is a letter in the word FOLLOW}\} = \{F, O, L, W\}$

$B = \{y: y \text{ is a letter in the word WOLF}\} = \{W, O, L, F\}$

The order in which the elements of a set are listed is not significant.

$\therefore$ $A = B$

Question 6:

From the sets given below, select equal sets:

$A = \{2, 4, 8, 12\}$, $B = \{1, 2, 3, 4\}$, $C = \{4, 8, 12, 14\}$, $D = \{3, 1, 4, 2\}$

$E = \{-1, 1\}$, $F = \{0, a\}$, $G = \{1, -1\}$, $H = \{0, 1\}$

Answer 6:

$A = \{2, 4, 8, 12\}$; $B = \{1, 2, 3, 4\}$; $C = \{4, 8, 12, 14\}$

$D = \{3, 1, 4, 2\}$; $E = \{-1, 1\}$; $F = \{0, a\}$

$G = \{1, -1\}$; $A = \{0, 1\}$

It can be seen that

$8 \in A$, $8 \notin B$, $8 \notin D$, $8 \notin E$, $8 \notin F$, $8 \notin G$, $8 \notin H$

$\Rightarrow A \neq B$, $A \neq D$, $A \neq E$, $A \neq F$, $A \neq G$, $A \neq H$

Also, $2 \in A$, $2 \notin C$

$\therefore A \neq C$

$3 \in B$, $3 \notin C$, $3 \notin E$, $3 \notin F$, $3 \notin G$, $3 \notin H$

$\therefore B \neq C$, $B \neq E$, $B \neq F$, $B \neq G$, $B \neq H$

$12 \in C$, $12 \notin D$, $12 \notin E$, $12 \notin F$, $12 \notin G$, $12 \notin H$

$\therefore C \neq D$, $C \neq E$, $C \neq F$, $C \neq G$, $C \neq H$

$4 \in D$, $4 \notin E$, $4 \notin F$, $4 \notin G$, $4 \notin H$

$\therefore D \neq E$, $D \neq F$, $D \neq G$, $D \neq H$

Similarly, $E \neq F$, $E \neq G$, $E \neq H$, $F \neq G$, $F \neq H$, $G \neq H$

The order in which the elements of a set are listed is not significant.

$\therefore B = D$ and $E = G$

Hence, among the given sets, $B = D$ and $E = G$.