Class -10th **Sub-Math**

1. Definitions of Real number, Rational number, irrational number.

-Do all the examples before exercise -1.3

-Prove that √5 is an irrational number.

-Prove that 3+2√5 is irrational.

-Prove that the following are irrational

1)1/√2 2) 7√5

Exercise 1.4

2. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

(i) $\frac{13}{3125}$ (ii)$\frac{17}{8}$ (iii) $\frac{64}{455}$(iv) $\frac{15}{1600}$ (v)$\frac{29}{343}$ (vi)$\frac{23}{2^3 5^2}$ (vii)$\frac{129}{2^2 5^7 7^5}$ (viii)$\frac{6}{15}$(ix)$\frac{35}{50}$ (x)$\frac{77}{210}$

Hint*, here find the prime factorization of denominator if factorization is in the form of $2^n \times 5^m$, then it is the terminating decimal Expression. If not it is non –terminating decimal.

Sol.$\frac{13}{3125} = \frac{13}{5*5*5*5*5}$

On multiplying by 2^5 in numerator and denominator ,we get

$\frac{13}{3125} = \frac{13*2*2*2*2*2}{5*2*5*2*5*2*5*2*5*2}$

$= \frac{13*32}{10*10*10*10*10} = \frac{416}{100000} = 0.00416$

ii) Do same as part (i) Ans.2.125

iii) Do same as part (i) Ans 0.009375

iv) Do same as part (i) Ans 0.115

viii) Do same as part (i) Ans 0.4

ix) Do same as part (i) Ans 0.7

3. The following real numbers have decimal expansions as given below. In each case , decide whether they are rational or not. If they are rational and of the form $\frac{P}{Q}$,then what we can say about the prime factors of q?

i) 43.123456789 ii) 0.120120012000120000 iii) $43.\overline{123456789}$

Sol. We know that ,a rational number has either terminating or non-terminating repeating decimal expansion.

(i) Here, 43.123456789 has terminating decimal expansion. So ,it represents a rational number.

i.e 4.3123456789= $\frac{43123456789}{100000000} = \frac{p}{q}$

Thus ,q=10^9, whose factors are $2^9 \times 5^9$.

ii) Here ,0.120120012000120000…………has non-terminating non-repeating decimal expansion. So it is an irrational number.

iii) Here $43.\overline{123456789}$ has non –terminating but repeating decimal expansion.so ,it is a rational number.

Let X= $43.\overline{123456789}$ ………..(i)

On multiplying by 1000000000 on both sides, we get 1000000000x =$43123456789.\overline{123456789}$ …. (ii)

On subtracting Eq. (i) from Eq (ii), we get

999999999x=43123456746

X=$\frac{43123456746}{999999999} = \frac{p}{q}$

Thus, q=999999999, which have factors other than 2 or 5.

Study material / Assignment for Unit 2nd

1. An algebraic expression p(x) of the form :

$P(x)=a_0+a_1x+a_2x^2+\ldots\ldots\ldots a_nx^n$.

Where $a_0, a_1, a_2 \ldots\ldots$ an are real numbers and all the indices (exponents)of X are non- negative integers is called a polynomial in X and the highest index (exponent) n is called the degree of the polynomial , if $a_n \neq 0$

$A_0, a_1x^1, a_2x^2, \ldots\ldots\ldots\ldots a_nx^n$ are called the terms of the polynomial and $a_0, a_1, a_2, \ldots\ldots\ldots\ldots a_n$ are called various coefficients of the polynomial p(x).

2. **A polynomial** in X is said to be in standard form when the terms are written either in increasing order or decreasing order of the indices of X in various terms.

For example:

(a) $2x^3 +3x^2+4x+5$

(b) $6x^2-5x+4$

(c) 4x-9

3. **Degree of the polynomial**: The highest power of the variable is called the degree of the polynomial . For example:

(a) $p(x)= 2x^3+6x^2+4x-5$,it is a polynomial of degree 3.

(b) $p(x)= 4x^2 – 5x +6$, it is a polynomial of degree 2.

(c) $p(x)= 7x+9$,it is a polynomial of degree 1.

4. **Type of the polynomials**: They can be classified on the basis of the degree.

(a) **Linear polynomial**: A polynomial of degree one is called a linear polynomial . it is of the form ax+b, $a\neq 0$, where a,b are real numbers.

For example: 2x +5, 4x-7,6x etc.

(b) **Quadratic polynomial** : A polynomial of degree two is called a quadratic polynomial .it is of the form ax^2+bx+c, $a\neq 0$,where a,b,c are real numbers.

For example: $9x^2-7x+1$, $2x^2+3x+4$, $2x^2-3$ etc.

(c) **cubic polynomial** : A polynomial of degree three is called a cubic polynomial . It is of the form $ax^3 +bx^2+cx +d$, $a\neq 0$, where a,b,c,d are real numbers.

For example- $2x^3+3x^2+4x +6$, $6x^3+4x^2-1$ etc.

Note- A cubic polynomial has at most three zeroes.

(d) Bi-quadratic polynomial: A polynomial of degree four is called a bi- quadratic polynomial .It is of the form

$Ax^4+bx^3+cx^2+dx +e$, $a\neq 0$, where a,b,c,d are real numbers.

For example- $2x^4+4x^3+3x^2+5x+9$, etc

Note- A biquadratic polynomial has at most four zeroes.

(e)**Value of polynomial**- ifp(x) is a polynomial in x and if k is any real constant ,then the real number obtained by replacing x by k in p(x) is called the value of p(x) at k and is denoted by p(k).

For example- p(x)- x^2+2x-8

$P(2)= 2^2+2(2)-8$

$P(2)= 4+4-8=8-8=0.$

(f) **zero of polynomial**- A real number k is called a zero of the polynomial ,p(x) if p(k) =0 . In other words , a zero of polynomial is the value of the variable for which the value of the polynomial becomes zero.

For example- $p(x)=x^2-7x+12$

$P(3)= 3^2-7(3)+12$

$=9-21+12=-12+12=0$

$P(4)= 4^2-7(4)+12 =16-28+12=-12+12=0$

Hence 3and 4 are the zeroes of p(x).

(g) The zero of the polynomial p(x) are precisely the x-coordinates of the points, where the graph of y=p(x) intersects the x-axis.

(h) If $\alpha and \beta$ are the two zeroes of the the quadratic polynomial ax^2+bx+c , then

(a) Sum of the zeroes $=\frac{-coefficient\ of\ X}{coefficient\ of\ X2}$

i.e...., $\alpha+\beta=\frac{-b}{a}$

(b) Product of the zeroes = $\frac{constant\ term}{coefficient\ of\ x2}$

i.e..,$\alpha - \beta = \frac{c}{a}$

(h) **To find the quadratic polynomial when the zeroes are given below**: if the zeroes of the quadratic polynomial are given then we can construct the quadratic polynomial .let $\alpha and\ \beta$ are the two zeroes of the quadratic polynomial then the required quadratic polynomial is

P(x) = X^2-(sum of zeroes) x + product of zeroes

i.e.... p(x) = X^2($\alpha + \beta)x + \alpha.\beta$

(I) The division algorithm states that given polynomial p(x) and any non- zero polynomial g(x) there exist polynomial q(x) and r(x) such that :

P(x)= g(x)× $q(x) + r(x)$

Where ,r(x) =0 or degree r(x) $< degreeq(x)$ .It is known as division algorithm.

Question 1:
Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) x^2-2x-8 (ii) $4s^2-4s+1$ (iii) $6x^2-3-7x$
(iv) $4u^2+8u$ (v) t^2-15 (vi) $3x^2-x-4$

Solution 1:
(i) $x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when x − 4 = 0 or x + 2 = 0, i.e., when x = 4 or x = −2

Therefore, the zeroes of x^2-2x-8 are 4 and −2.

Sum of zeroes = $4-2=2=\frac{(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2-4s+1=(2s-1)^2$

The value of $4s^2-4s+1$ is zero when 2s − 1 = 0, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2-4s+1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2}+\frac{1}{2}=1\frac{(-4)}{4}=\frac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2}$

Product of zeroes $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of } s^2}$

(iii) $6x^2-3-7x=6x^2-7x-3=(3x+1)(2x-3)$

The value of $6x^2-3-7x$ is zero when 3x + 1 = 0 or 2x − 3 = 0, i.e.,

$x=\frac{-1}{3}$ or $x=\frac{3}{2}$

Therefore, the zeroes of $6x^2-3-7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$

Sum of zeroes $= \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $4u^2 + 8u = 4u^2 + 8u + 0$
$= 4u(u+2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and −2.

Sum of zeroes $= 0 + (-2) = -2 = \frac{(-8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes $= 0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(v)

$t^2 - 15$

$= t^2 = 0t - 15$

$= (t - \sqrt{15})(t + \sqrt{15})$

The value of $t^2 - 15$ is zero when $t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$, i.e., when $t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are and $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes $= \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2}$

Product of zeroes $= (\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(vi) $3x^2 - x - 4$

The value of 3x2 − x − 4 is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e., when $x = \frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and −1.

Sum of zeroes = $\frac{4}{3}+(-1)=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=\frac{4}{3}+(-1)=\frac{-4}{3}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$ (ii) $\sqrt{2},\frac{1}{3}$ (iii) 0, 5 (iv) 1, 1 (v) $-\frac{1}{4},\frac{1}{4}$

(vi) 4, 1

Solution 2:

(i) $\frac{1}{4},-1$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$\alpha+\beta=\frac{1}{4}=\frac{-b}{a}$

$\alpha\beta=-1=\frac{-4}{4}=\frac{c}{a}$

If a = 4, then b = −1, c = -4

Therefore, the quadratic polynomial is $4x^2-x-4$.

(ii) $\sqrt{2},\frac{1}{3}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$\alpha+\beta=\sqrt{2}=\frac{3\sqrt{2}}{3}=\frac{-b}{a}$

$\alpha\beta=\frac{1}{3}=\frac{c}{a}$

If a = 3, then b = $-3\sqrt{2}$, c = 1

Therefore, the quadratic polynomial is $3x^2-3\sqrt{2}x+1$.

(iii) 0, $\sqrt{5}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$

If a = 1, then b = 0, c = $\sqrt{5}$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

(iv) 1, 1

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If a = 1, then b = -1, c = 1

Therefore, the quadratic polynomial is $x^2 - x + 1$.

(v) $-\frac{1}{4}, \frac{1}{4}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = \frac{-1}{4} = \frac{-b}{a}$$

$$\alpha \times \beta = \frac{1}{4} = \frac{c}{a}$$

If a = 4, then b = 1, c = 1

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

(vi) 4, 1

Let the polynomial be $ax^2 + bx + c$.

$$\alpha + \beta = 4 = \frac{4}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If a = 1, then b = -4, c =1

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.

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