SYSTEM OF MEASUREMENT OF ANGLES

There are two system which are commonly use for measuring angles

(1) Sexagesimal or English system :

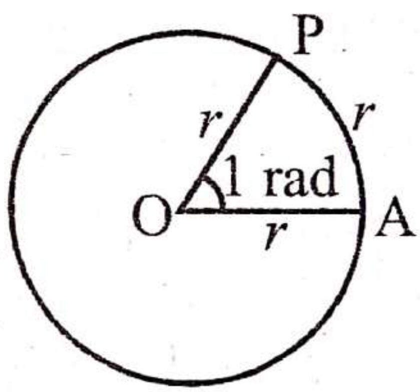
$$1 \text{ right angle} = 90 \text{ degree } (= 90^\circ)$$

$$1^\circ = 60 \text{ minutes } (= 60')$$

$$1' = 60 \text{ second } (= 60'')$$

(2) Circular system : The measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle is called one radian.

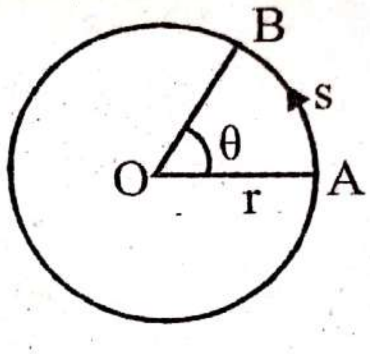
Consider a circle of radius r having centre at O. Let A be a point on the circle. Now cut off an arc AP whose length is equal to the radius r of the circle. Then by the definition the measure of $\angle AOP$ is 1 radian $(= 1^c)$



$$\pi \text{ radians} = 180^\circ$$

RELATION BETWEEN ARC, RADIUS AND CENTRAL ANGLE

If s is the length of an arc of a circle of radius r, then the angle θ (in radians) subtended by this arc at the centre of the circle is given by $\theta = \frac{s}{r}$ or $s = r\theta$.

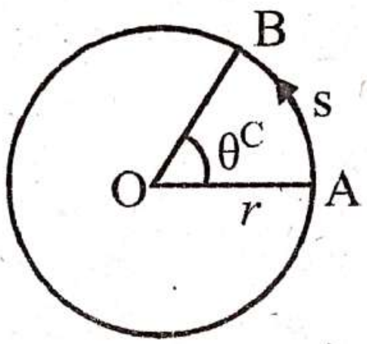


i.e., Arc = radius × angle in radians

AREA OF A SECTOR

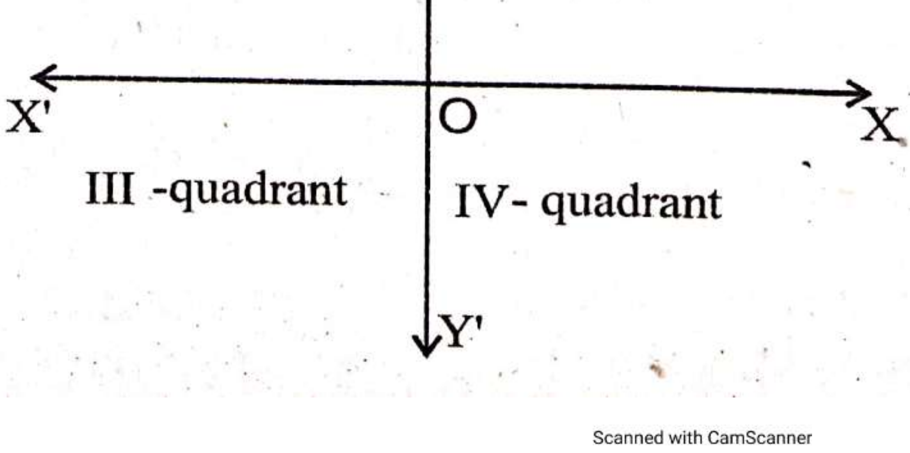
OAB be a sector having central angle θ (in radians) and radius r.

Area of the sector OAB is given by $\frac{\pi}{2} r^2 \theta$.



SOME USEFUL TERMS

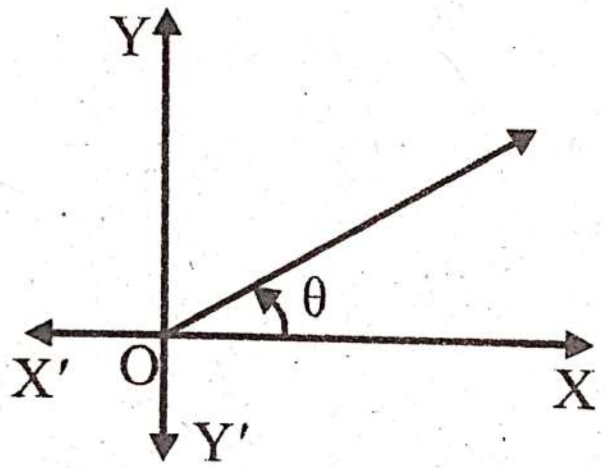
Quadrants : Let X′OX and YOY′ be two lines at right angles in the plane of the paper X′ OX is horizontal line and YOY′ is vertical line. These lines divide the plane of the paper into four equal parts, each of which is known a quadrant.



The lines X′OX and YOY′ are known as x-axis and y-axis respectively. These two lines taken together are known as the coordinate axes. The regions XOY, YOX′, X′OY′ and Y′OX are known as the first, the second, the third and the fourth quadrant respectively.

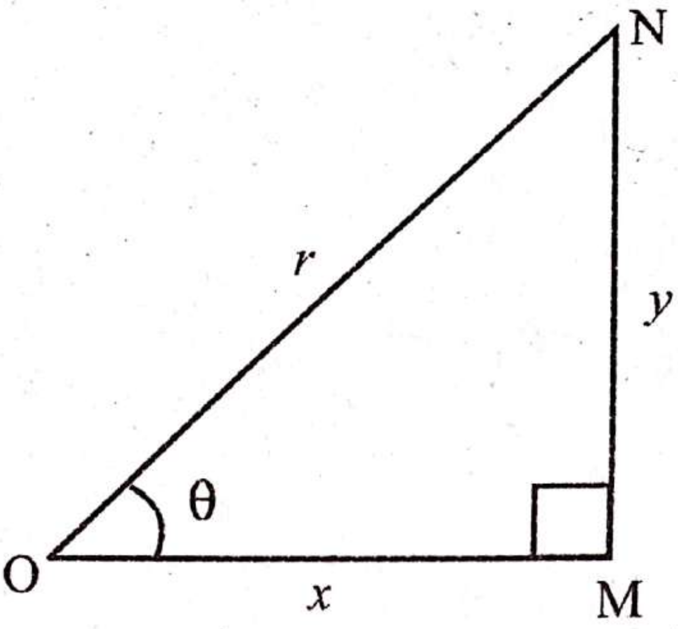
Angle In Standard Position : An angle is said to be in standard position if its vertex coincides with the origin O and the initial side coincides with OX.

Here θ is the angle in standard position.



TRIGONOMETRICAL RATIOS

In the right angled triangle OMN, we have base (OM) = x, perpendicular (NM) = y and hypotenuse (ON) = r, then we define the following trigonometric ratios which are also known as trigonometric functions.



$$\sin\theta = \frac{P}{H} = \frac{y}{r}, \quad \cos\theta = \frac{B}{H} = \frac{x}{r}, \quad \tan\theta = \frac{P}{B} = \frac{y}{x}.$$

$$\cot\theta = \frac{B}{p} = \frac{x}{y}, \sec\theta = \frac{H}{B} = \frac{r}{x}, \text{cosec}\,\theta = \frac{H}{p} = \frac{r}{y}$$

FUNDAMENTAL TRIGONOMETRIC IDENTITIES

(i) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(ii) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

(iii) $\sin\theta = \dfrac{1}{\text{cosec}\,\theta} \Rightarrow \sin\theta.\cos\theta = 1$

(iv) $\cos\theta = \dfrac{1}{\sec\theta} \Rightarrow \cos\theta\sec\theta = 1$

(v) $\cot\theta = \dfrac{1}{\tan\theta} \Rightarrow \tan\theta.\cot\theta = 1$

(vi) $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta \Rightarrow \cos^2\theta = 1 - \sin^2\theta$

(vii) $\sec^2\theta - \tan^2\theta = 1 \Rightarrow \sec^2\theta = 1 + \tan^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$

$$\Rightarrow \quad \sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

(viii) $\text{cosec}^2\theta - \cot^2\theta = 1 \Rightarrow \text{cosec}^2\theta = 1 + \cot^2\theta$

$$\Rightarrow \quad \cot^2\theta = \text{cosec}^2\theta - 1$$

$$\Rightarrow \quad \text{cosec}\,\theta - \cot\theta = \frac{1}{\text{cosec}\,\theta + \cot\theta}$$

EXAMPLE 1 :

If cosec A + cot A = 11/2, then find the value of tan A.

Sol. $\text{cosec}\,A + \cot A = 11/2$ ...(1)

$$\Rightarrow \quad \frac{1}{\text{cosec}\,A + \cot A} = \frac{2}{11}$$

$$\Rightarrow \quad \text{cosec}\,A - \cot A = \frac{2}{11} \qquad \text{...(2)}$$

$$(1) - (2) \Rightarrow 2\cot A = \frac{11}{2} - \frac{2}{11} = \frac{117}{22} \Rightarrow \tan A = \frac{44}{117}$$

EXAMPLE 2:

Find the value of $\dfrac{\cos\theta}{1-\tan\theta}+\dfrac{\sin\theta}{1-\cot\theta}$

Sol. $$\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}=\frac{\cos\theta}{1-\dfrac{\sin\theta}{\cos\theta}}+\frac{\sin\theta}{1-\dfrac{\cos\theta}{\sin\theta}}$$

$$=\frac{\cos^2\theta}{\cos\theta-\sin\theta}-\frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$=\frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}=\cos\theta+\sin\theta$$

EXAMPLE 3:

Find the value of $\tan^2\theta\sec^2\theta(\cot^2\theta-\cos^2\theta)$.

Sol. $\tan^2\theta\sec^2\theta(\cot^2\theta-\cos^2\theta)$

$$=\sec^2\theta(\tan^2\theta\cot^2\theta-\tan^2\theta\cos^2\theta)$$

$$=\sec^2\theta\left(1-\frac{\sin^2\theta}{\cos^2\theta}\cos^2\theta\right)=\sec^2\theta(1-\sin^2\theta)$$

$$=\sec^2\theta.\cos^2\theta=1$$

SIGN OF THE TRIGONOMETRICAL RATIOS

Sign of a trigonometrical ratio depends on the quadrant in which the terminal side of the angle lies.

In First quadrant : $x>0, y>0$

$$\Rightarrow \quad \sin\theta = \frac{y}{r} > 0,\ \cos\theta = \frac{x}{r} > 0,\ \tan\theta = \frac{y}{x} > 0,$$

$$\operatorname{cosec}\theta = \frac{r}{y} > 0,\ \sec\theta = \frac{r}{x} > 0 \text{ and } \cot\theta = \frac{x}{y} > 0$$

Thus, in the first quadrant all trigonometric functions are positive

In Second quadrant : $x<0, y>0, r>0$

$$\Rightarrow \quad \sin\theta = \frac{y}{r} > 0,\ \cos\theta = \frac{x}{r} < 0,\ \tan\theta = \frac{y}{x} < 0,$$

$$\operatorname{cosec}\theta = \frac{r}{y} > 0,\ \sec\theta = \frac{\mathrm{r}}{\mathrm{x}} < 0 \text{ and } \cot\theta = \frac{x}{y} < 0$$

Thus, in the second quadrant sin and cosec function are positive and all others negative

In Third quadrant : $x<0, y<0, r>0$

$$\sin\theta = \frac{y}{r} < 0,\ \cos\theta = \frac{x}{r} < 0,\ \tan\theta = \frac{y}{x} > 0,\ \operatorname{cosec}\theta = \frac{r}{y} < 0,$$

$$\sec\theta = \frac{r}{x} < 0 \text{ and } \cot\theta = \frac{x}{y} > 0$$

Thus, in the third quadrant all trigonometric functions are negative except tangent and cotangent.

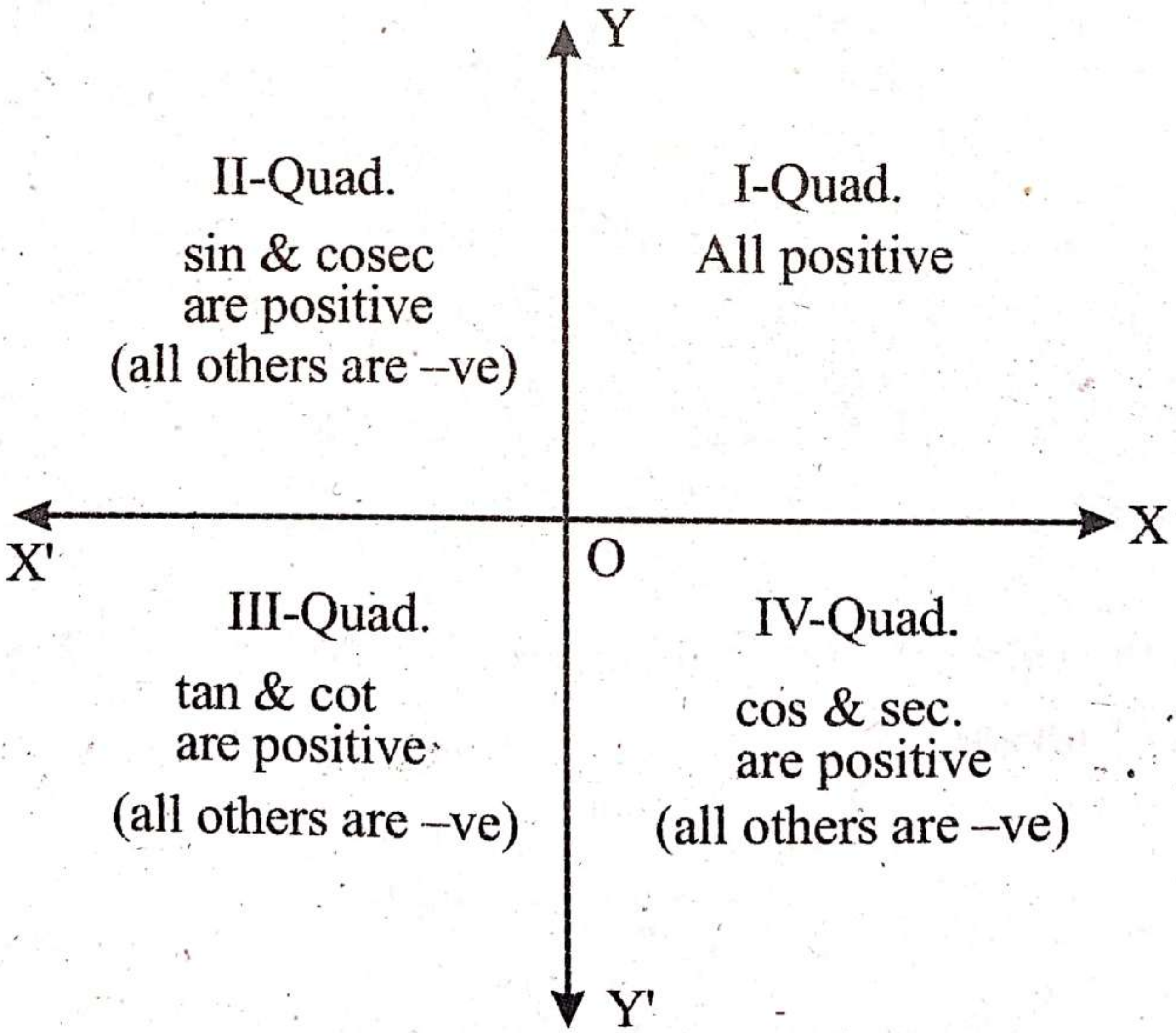
In Fourth quadrant : $x>0, y<0, r>0$

$$\Rightarrow \quad \sin\theta = \frac{y}{r} < 0,\ \cos\theta = \frac{x}{r} > 0,\ \tan\theta = \frac{y}{x} < 0,\ \mathrm{cosec}\,\theta = \frac{r}{y} < 0,$$

$$\sec\theta = \frac{r}{x} > 0 \text{ and } \cot\theta = \frac{x}{y} < 0$$

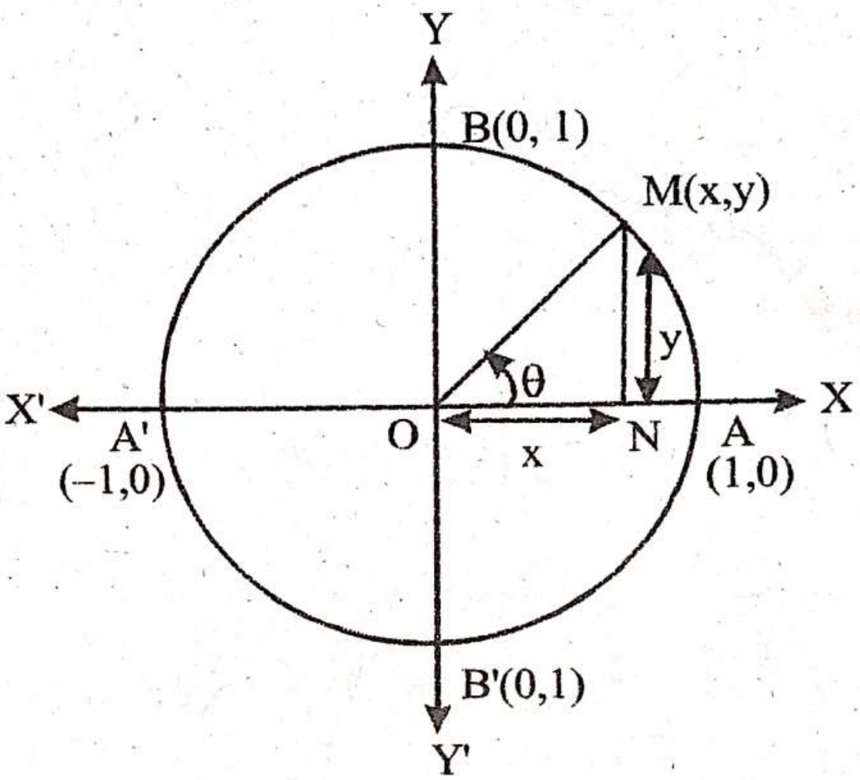
Thus, in the fourth quadrant all trigonometric functions are negative except cos and sec.

In Brief,



VARIATIONS IN VALUES OF TRIGONOMETRIC FUNCTIONS IN DIFFERENT QUADRANTS

Let X' OX and YOY' be the coordinate axes. Draw a circle with centre at origin O and radius unity.



Let M (x, y) be a point on the circle such that $\angle AOM = \theta$ then $x = \cos\theta$ and $y = \sin\theta$

II - Quadrant

$\sin\theta \longrightarrow$ Decrease from 1 to 0
$\cos\theta \longrightarrow$ Decrease from 0 to -1
$\tan\theta \longrightarrow$ Increase from $-\infty$ to 0
$\cot\theta \longrightarrow$ Decrease from 0 to $-\infty$
$\sec\theta \longrightarrow$ Increase from $-\infty$ to -1
$\text{cosec}\,\theta \longrightarrow$ Increase from 1 to ∞

I - Quadrant

$\sin\theta \longrightarrow$ Increase from 0 to 1
$\cos\theta \longrightarrow$ Decrease from 1 to 0
$\tan\theta \longrightarrow$ Increase from 0 to ∞
$\cot\theta \longrightarrow$ Decrease from ∞ to 0
$\sec\theta \longrightarrow$ Increase from 1 to ∞
$\text{cosec}\,\theta \longrightarrow$ Decrease from ∞ to 1

III - Quadrant

$\sin\theta \longrightarrow$ Decrease from 0 to -1
$\cos\theta \longrightarrow$ Increase from -1 to 0
$\tan\theta \longrightarrow$ Increase from 0 to ∞
$\cot\theta \longrightarrow$ Decrease from ∞ to 0
$\sec\theta \longrightarrow$ Decrease from -1 to $-\infty$
$\text{cosec}\,\theta \longrightarrow$ Increase from $-\infty$ to -1

IV - Quadrant

$\sin\theta \longrightarrow$ Increase from -1 to 0
$\cos\theta \longrightarrow$ Increase from 0 to 1
$\tan\theta \longrightarrow$ Increase from $-\infty$ to 0
$\cot\theta \longrightarrow$ Decrease from 0 to $-\infty$
$\sec\theta \longrightarrow$ Decrease from ∞ to 1
$\text{cosec}\,\theta \longrightarrow$ Decrease from -1 to $-\infty$

Thus, $-1 \le \cos\theta \le 1$ and $-1 \le \sin\theta \le 1$ for all values of θ