PHYSICS – XI

Unit – I Elementary Maths:-

Part – I

I) Quadratic Equations:-

The general quadratic equation is given by:-

$$\boxed{ax^2 + bx + c = 0}$$

i) Roots of Q. Eqn:- $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

ii) Discriminant :- $D = b^2 - 4ac$

iii) Nature of roots:- a) if $D > 0$, Roots are real and unequal.

b) if $D = 0$, Roots are real and equal.

c) if $D < 0$, No Real Roots.

iv) Sum of Roots:- $\alpha + \beta = -b/a$

v) Product of Roots:- $\alpha \cdot \beta = c/a$

vi) Formation of Q. Eqn:- $\boxed{x^2 - S.x + P = 0}$

where $S = \alpha + \beta$ and $P = \alpha \cdot \beta$

Questions:- 1) Find the roots of :- $px^2 + qx + r = 0$

Sol:- Here $a = p$, $b = q$, $c = r$

$$x = \frac{-q \pm \sqrt{q^2 - 4pr}}{2p} \text{ Ans.}$$

(2) Find the roots of the Q. Eqn:- $x - \frac{1}{x} = 0$

Sol:- $x - \frac{1}{x} = 0$

$\frac{x^2 - 1}{x} = 0$

or $x^2 - 1 = 0$

or $x^2 = 1 \Rightarrow x = \pm 1$

$\therefore x = +1, -1$ Ans.

(3) Find the roots of the Q. Eqns:-

$x^2 - 2ax + a^2 - b^2 = 0$

Sol:- $x = \dfrac{2a \pm \sqrt{4a^2 - 4a^2 + 4b^2}}{2}$

$x = \dfrac{2a \pm 2b}{2}$ or $\boxed{x = a \pm b}$ Ans.

(4) $2x^2 - 13x + 5 = 0$

Sol:- $x = \dfrac{13 \pm \sqrt{129}}{4}$ or $\boxed{x = \dfrac{13 \pm \sqrt{129}}{4}}$ Ans.

Q:- If α, β are the roots of Q. Equ $ax^2 + bx + c = 0$ find the values of the following:-

(i) $\alpha + \beta$ (ii) $\alpha - \beta$ (iii) $\alpha^2 + \beta^2$ (iv) $\alpha^2 - \beta^2$ (v) $\alpha \cdot \beta$

Solutions:- (i) $\alpha + \beta = -b/a$ Ans

(ii) $\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$\alpha - \beta = \sqrt{\dfrac{b^2}{a^2} - \dfrac{4c}{a}}$

$\boxed{\alpha - \beta = \sqrt{\dfrac{b^2 - 4ac}{a}}}$ Ans.

(iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\alpha^2 + \beta^2 = \dfrac{b^2}{a^2} - \dfrac{2c}{a}$

or $\boxed{\alpha^2 + \beta^2 = \dfrac{b^2 - 2ac}{a^2}}$ Ans

(iv) $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta)$

$\alpha^2 - \beta^2 = (-b/a)\left(\sqrt{\dfrac{b^2 - 4ac}{a}}\right)$

$\boxed{\alpha^2 - \beta^2 = \dfrac{-b\sqrt{b^2 - 4ac}}{a^2}}$ Ans.

(v) $\alpha \cdot \beta = c/a$ Ans.

(II) BINOMIAL THEOREM

(i) For 'n' any −ve index or fraction:-

$$(1+x)^n = 1 + \frac{n}{1!}\cdot x + \frac{n(n-1)}{2!}\cdot x^2 + \frac{n(n-1)(n-2)}{3!}\cdot x^3 + \cdots$$

Here $|x| < 1$

(ii) For 'n' any +ve index:-

$$(x+a)^n = x^n + \frac{n\cdot x^{n-1}}{1!}a + \frac{n(n-1)x^{n-2}}{2!}a^2 + \cdots + a^n$$

Q:- Expand the $(1+x)^{-2}$ binomially:-

Sol:- $(1+x)^{-2} = 1 + (-2)x + \frac{(-2)(-2-1)}{2!}x^2 + \frac{(-2)(-2-1)(-2-2)}{3!}x^3 + \cdots$

$(1+x)^{-2} = 1 - 2x + \frac{6}{2!}x^2 - \frac{24}{3!}x^3 + \cdots$ to ∞

$= 1 - 2x + 3x^2 - 4x^3 + \cdots$ to ∞ Ans.

(b) $\sqrt{26}$ expand upto 4 decimal places binomially:

Sol:- $\sqrt{26} = \sqrt{25+1}$ or $\sqrt{26} = (25+1)^{1/2}$

$\therefore \sqrt{26} = (25)^{1/2}\left(1+\frac{1}{25}\right)^{1/2}$

$= 5 + (1+0.04)^{1/2}$

$= 5\left[1 + \frac{1}{2}(0.04) + \frac{\frac{1}{2}(-1/2)}{2!}(0.04)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-3/2)}{3!}(0.04)^3 + \cdots\right]$

neglecting the terms containing the powers such as $(0.04)^2, (0.04)^5$ etc we get:-

$\sqrt{26} = [1 + 0.02 - \frac{1}{8}(0.00016) + \frac{1}{16}(0.000046) + \cdots$

$= 5[1 + 0.02 - 0.0002 + 0.000004 + \cdots] = 5.0990$ Ans

Q:- The acceleration due to gravity of body at a certain height above the surface of earth is given by $g' = g\cdot\frac{R^2}{(R+h)^2}$. If $h << R$, then prove that $g' = g\left(1 - \frac{2h}{R}\right)$. where $R = 6.4\times10^6$ m.

Sol:- $g' = \frac{g\cdot R^2}{(R+h)^2} = \frac{g\cdot R^2}{R^2\left(1+\frac{h}{R}\right)^2} \Rightarrow g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

or $g' = g\left[1+\frac{h}{R}\right]^{-2}$ opening binomially we get:-

$g' = g\left[1 + \frac{-2h}{R} + \frac{-2(-2-1)h^2}{2\times R^2} + \cdots\right]$

$g' = g\left[1 - \frac{2h}{R} + \frac{3h^2}{R^2} + \cdots\right]$

Since $h << R$, so the higher powers of 'h' can be neglected as compared to R:-

$\therefore g' = g\left[1 - \frac{2h}{R}\right]$ Ans

Q. Expand the following functions binomially:-
(i) $(1+x)^{1/3}$ 4 terms (ii) $(1+x)^{-1}$ 6 terms (iii) $(1+x)^{2/3}$ 4 terms (iv) $\sqrt{10}$ 5 decimal

Sols:- $(1+x)^{1/3} = 1 + \frac{\frac{1}{3}x}{1!} + \frac{\frac{1}{3}(\frac{1}{3}-1)x^2}{2!} + \frac{\frac{1}{3}(\frac{1}{3}-1)(\frac{1}{3}-2)x^3}{3!}$

$\Rightarrow (1+x)^{1/3} = 1 + \frac{1}{3}x - \frac{2x^2}{9\times 2} + \frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})x^3}{3\times 2\times 1}$

$\boxed{(1+x)^{1/3} = 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3}$ Ans.

(ii) $(1+x)^{-1} = 1 + \frac{-1}{1!}x + \frac{(-1)(-2)}{2!}x^2 + \frac{(-1)(-2)(-3)x^3}{3!} + \frac{(-1)(-2)(-3)(-4)x^4}{4!} + \frac{(-1)(-2)(-3)(-4)(-5)x^5}{5!}$

$\boxed{(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - x^5}$ Ans

(iii) $(1+x)^{2/3} = 1 + \frac{2}{3}x + \frac{\frac{2}{3}(\frac{2}{3}-1)x^2}{2!} + \frac{\frac{2}{3}(\frac{2}{3}-1)(\frac{2}{3}-2)x^3}{3\times 2\times 1} + \frac{\frac{2}{3}(\frac{2}{3}-1)(\frac{2}{3}-2)(\frac{2}{3}-3)x^4}{4\times 3\times 2\times 1}$

$\Rightarrow (1+x)^{3/2} = 1 + \frac{2}{3}x + (\frac{2}{3})(-\frac{1}{3})(\frac{x^2}{2}) + (\frac{2}{3})(\frac{-1}{3})(-\frac{4}{3})\frac{x^3}{3\times 2} + \frac{(\frac{2}{3})(\frac{-1}{3})(-\frac{4}{3})(-\frac{7}{3}).x^4}{4\times 3\times 2\times 1}$

$\Rightarrow (1+x)^{3/2} = 1 + \frac{2}{3}x - \frac{x^2}{9} + \frac{4x^3}{81} - \frac{7x^4}{243}$

Or $\boxed{(1+x)^{3/2} = 1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 - \frac{7}{243}x^4}$ Ans

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